Beyond Single Stream
with the
Schrödinger method

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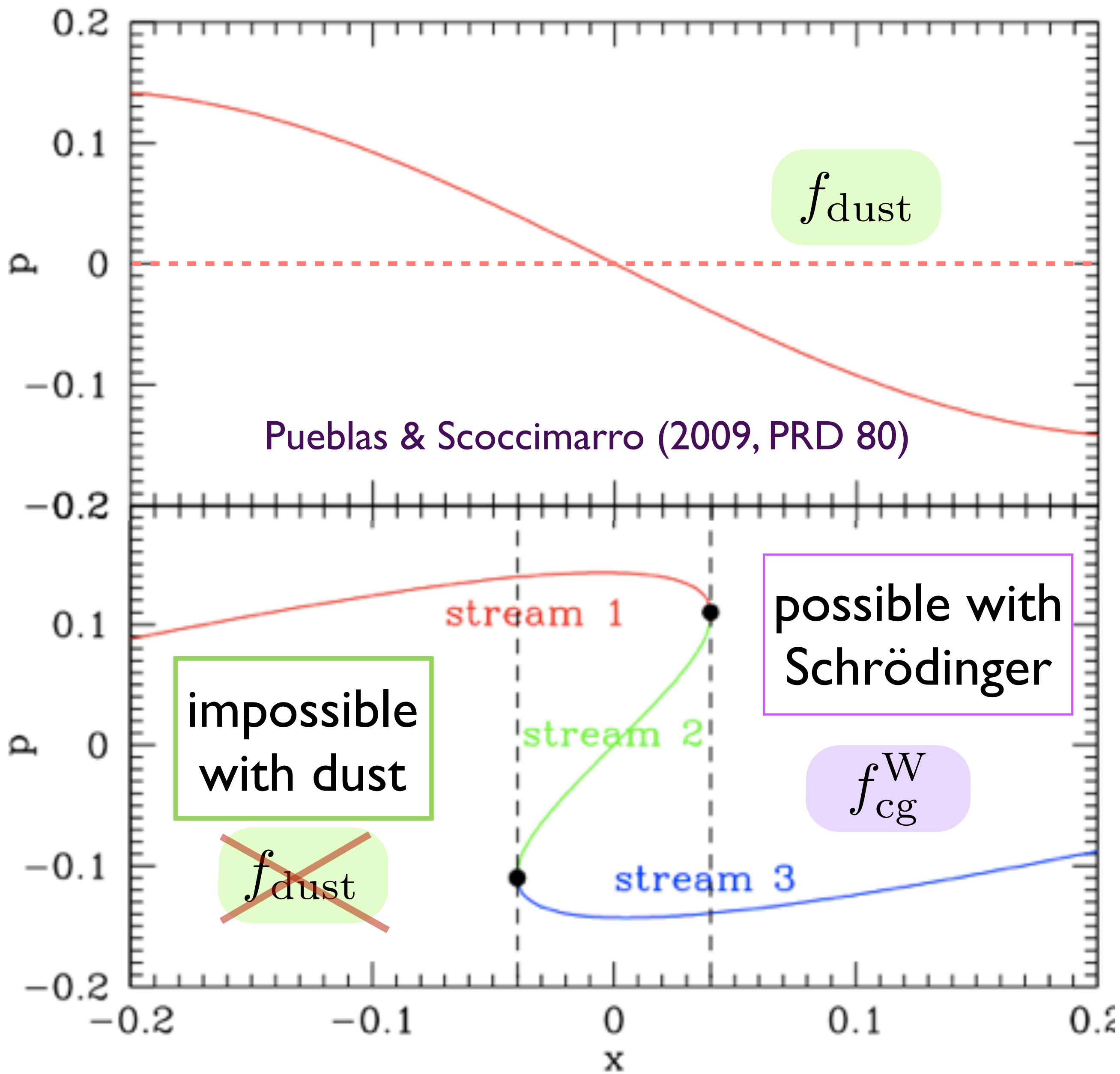
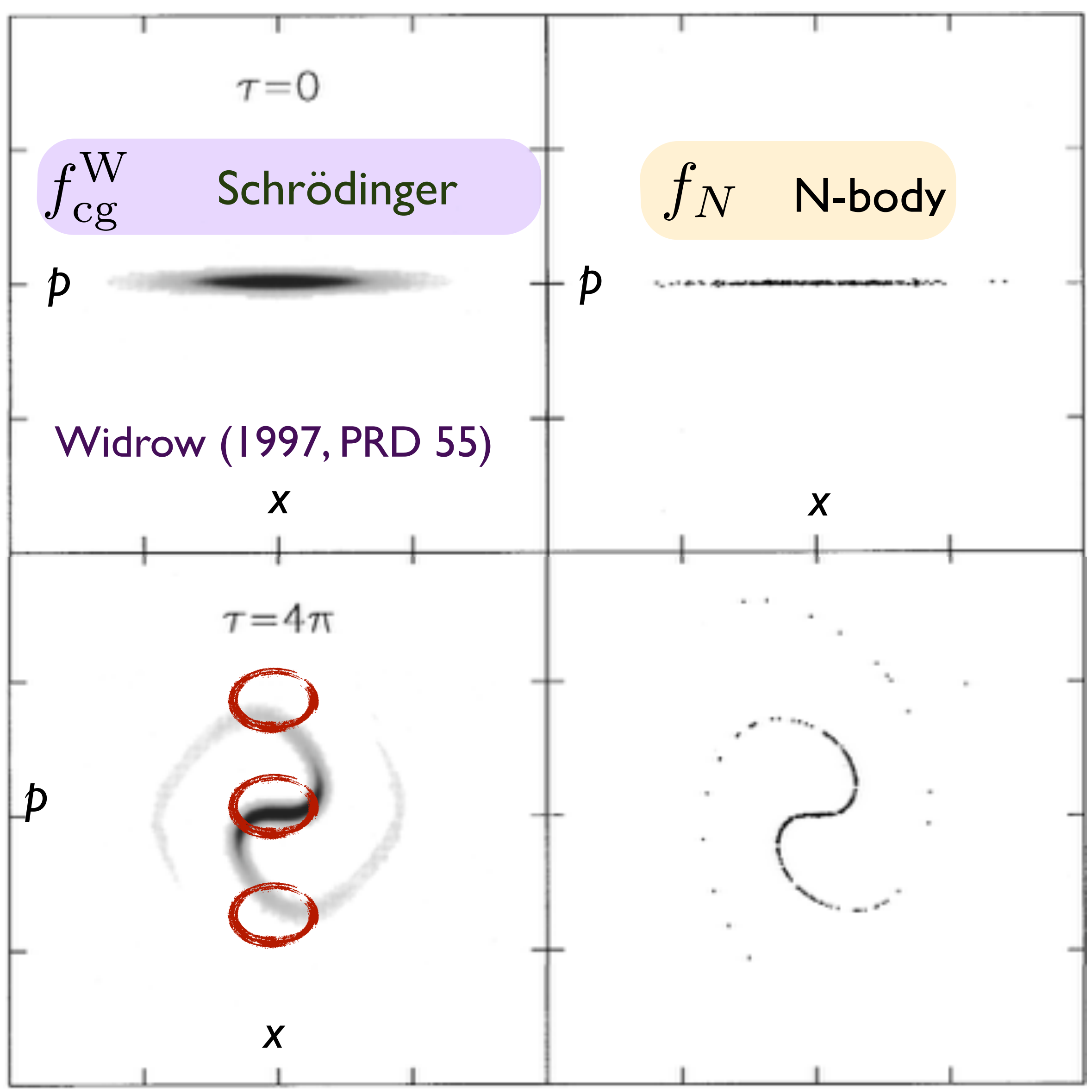
Description of CDM

- **phase-space distribution function** *f(x,p)* describes at each point *x* in space
  - number density & distribution of momenta *p*
- widespread standard **dust** ansatz for *f*
  - breaks down at **shell-crossing,** no virialization
- want analytical tool to study condensation of bound structures (halos)
  - allow for **multi-streaming and virialization**

Schrödinger method: Widrow & Kaiser (1993, ApJ 416)





Description of CDM

phase space distribution function

- **N-body**: non-relativistic, gravitational interaction
- **continuous**: ensemble average, drop 1/N collision terms

$$f_N = \sum_i \delta_D(\boldsymbol{x} - \boldsymbol{x}_i)\delta_D(\boldsymbol{p} - \boldsymbol{p}_i) \rightarrow f$$

Vlasov - Poisson equation

$$\partial_\tau f(\boldsymbol{x}, \boldsymbol{p}, \tau) = -\frac{\boldsymbol{p}}{am}\boldsymbol{\nabla}_x f + am\boldsymbol{\nabla}_x V \boldsymbol{\nabla}_p f$$

$$\Delta V(\boldsymbol{x}, \tau) = \frac{4\pi G m}{a}\left(n(\boldsymbol{x}, \tau) - \langle n \rangle\right)$$

as complicated as N-body simulation

induces nonlinearity

Hierarchy of Moments $M^{(n)}(\boldsymbol{x}) = \int d^3p\, p_{i_1} \ldots p_{i_n} f$

- density n(x): $M^{(0)} = n(\boldsymbol{x})$, velocity **v**(x): $M^{(1)} = n\boldsymbol{v}(\boldsymbol{x})$
- velocity dispersion **σ**(x): $M^{(2)} = n\left(\boldsymbol{v}\boldsymbol{v} + \boldsymbol{\sigma}\right)(\boldsymbol{x}), \ldots$

$$\partial_t M^{(n)} = -\frac{1}{a^2 m}\nabla \cdot M^{(n+1)} - m\nabla V \cdot M^{(n-1)}$$

infinite coupled hierarchy

Dust model

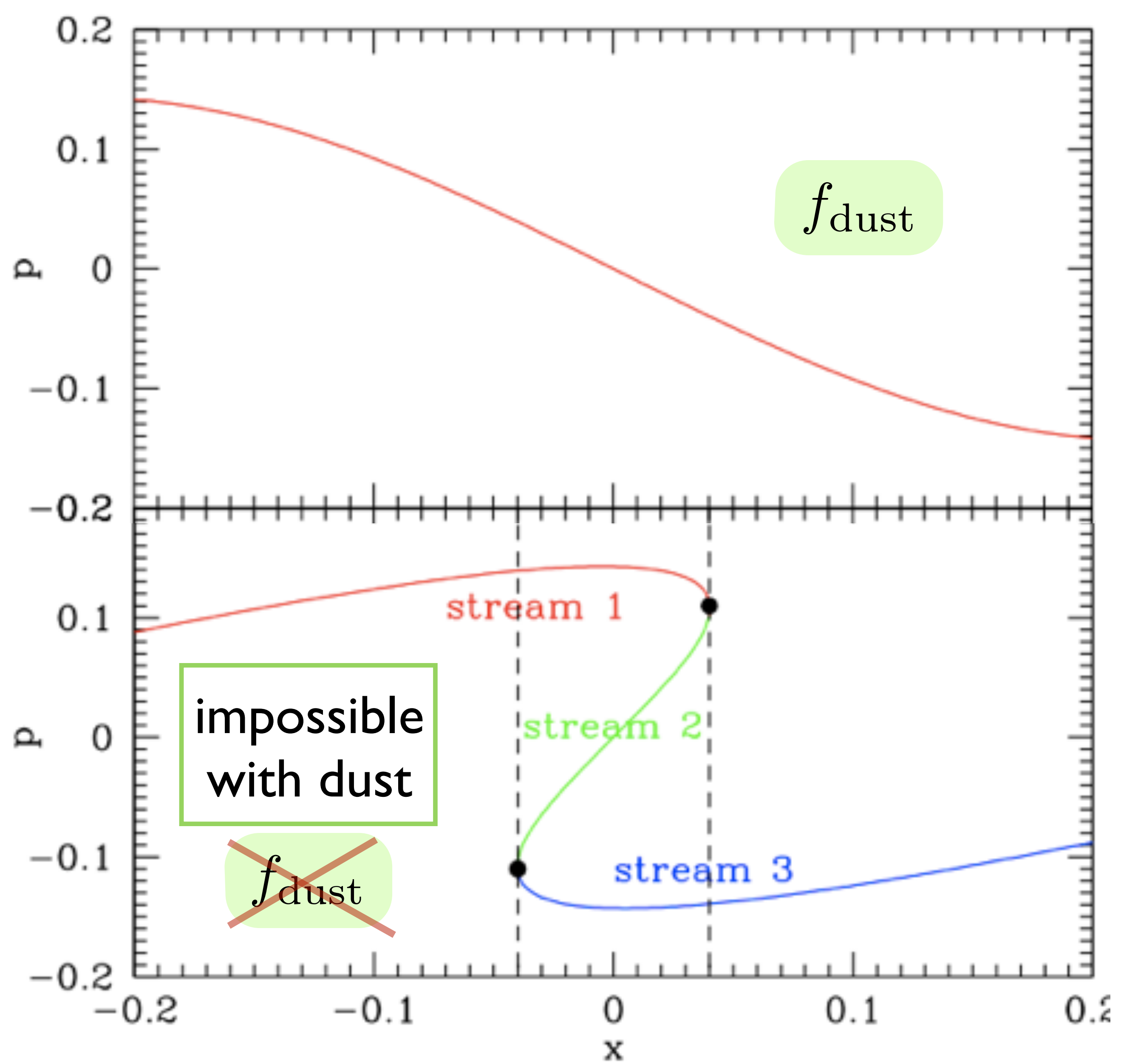
dust model

- only consistent **truncation of hierarchy**
- pressureless fluid: density and velocity

$$f_{\text{dust}}(\boldsymbol{x},\boldsymbol{p},\tau) = n(\boldsymbol{x},\tau)\delta_D^{(3)}(\boldsymbol{p} - \boldsymbol{\nabla}\phi(\boldsymbol{x},\tau))$$

Continuity $\partial_\tau n = -\frac{1}{am}\boldsymbol{\nabla}(n\boldsymbol{\nabla}\phi)$

Euler $\partial_\tau \phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV$



- no velocity dispersion or higher cumulants
- **shell-crossing singularities**

Schrödinger method

Schrödinger method

- Coarse-grained Wigner function, constructed from self-gravitating field $\psi = \sqrt{n} \exp\left(\frac{i}{\hbar}\phi\right)$
- amplitude and phase of wave function

$$f_{cg}^{W}(\boldsymbol{x},\boldsymbol{p}) = \int \frac{d^3x' d^3p'}{(\pi\sigma_x\sigma_p)^3} \exp\left[-\frac{(\boldsymbol{x}-\boldsymbol{x}')^2}{2\sigma_x^2} - \frac{(\boldsymbol{p}-\boldsymbol{p}')^2}{2\sigma_p^2}\right] \int \frac{d^3\tilde{x}}{(2\pi\hbar)^3} \exp\left[2\frac{i}{\hbar}\boldsymbol{p}'\cdot\tilde{\boldsymbol{x}}\right] \psi(\boldsymbol{x}'-\tilde{\boldsymbol{x}})\bar{\psi}(\boldsymbol{x}'+\tilde{\boldsymbol{x}})$$

$$\sigma_x\sigma_p \gtrsim \hbar/2$$

Continuity $\partial_\tau n = -\frac{1}{am}\boldsymbol{\nabla}(n\boldsymbol{\nabla}\phi)$

quantum potential

Euler $\partial_\tau \phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV + \frac{\hbar^2}{2am}\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)$

- macroscopic quantities: coarse-grained density & mass-weighted velocity

$$\bar{n}(\boldsymbol{x}) = \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right] n(\boldsymbol{x}) \qquad \bar{\boldsymbol{v}}(\boldsymbol{x}) = \frac{1}{am\bar{n}(\boldsymbol{x})} \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right] (n\boldsymbol{\nabla}\phi)(\boldsymbol{x})$$

- **closure of hierarchy**: all cumulants can be calculated self-consistently
- **regularizes shell-crossing singularities**

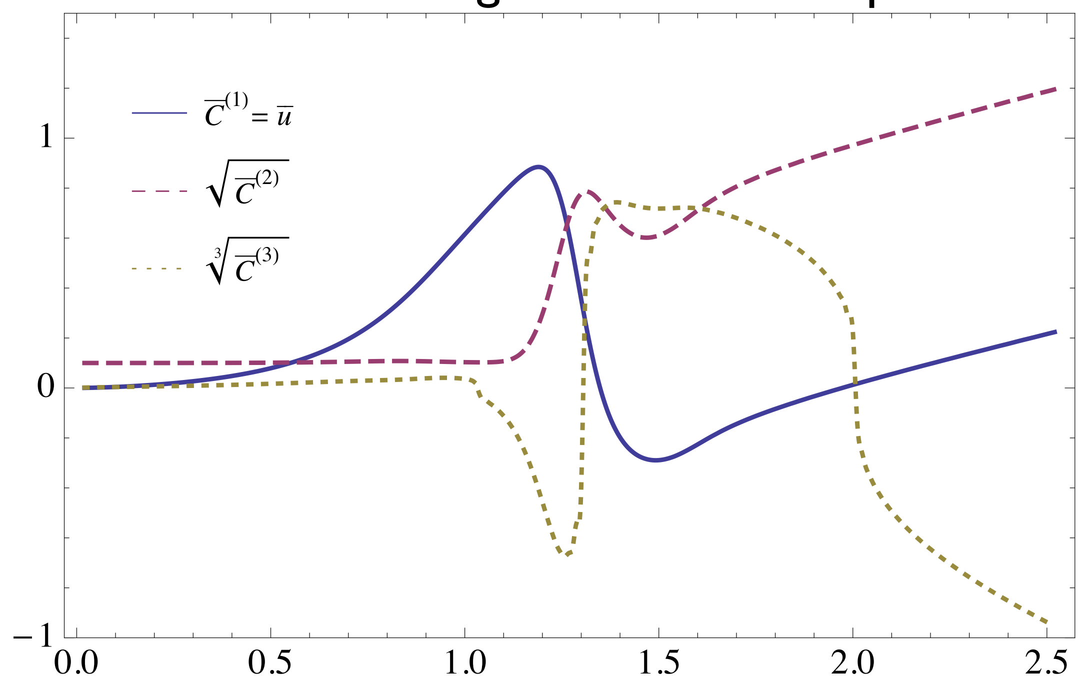
Features of Schrödinger Method

Multi-streaming

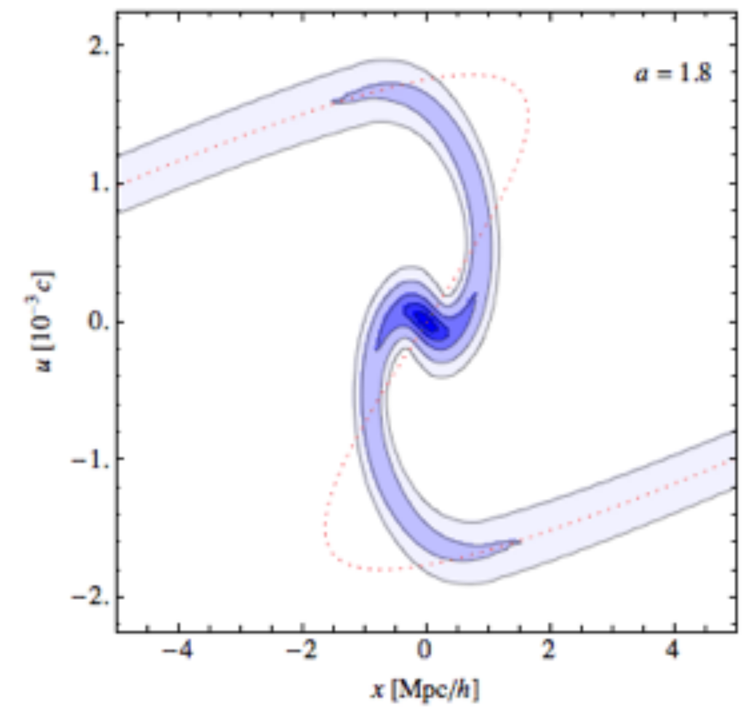
❌ dust model:
fails at shell-crossing

✅ Schrödinger method:
beyond shell-crossing

after shell-crossing: all cumulants important



phase space density

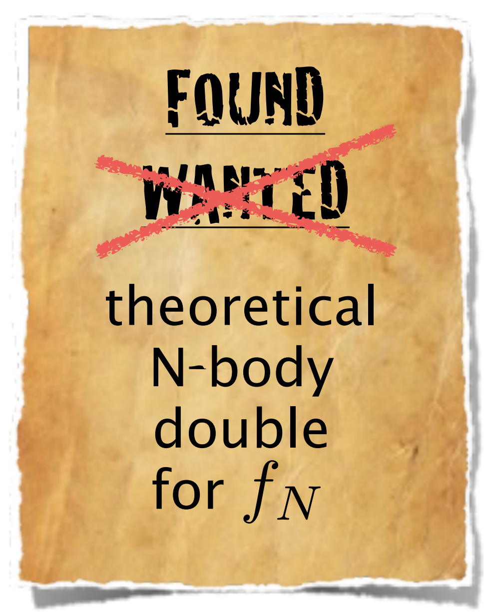


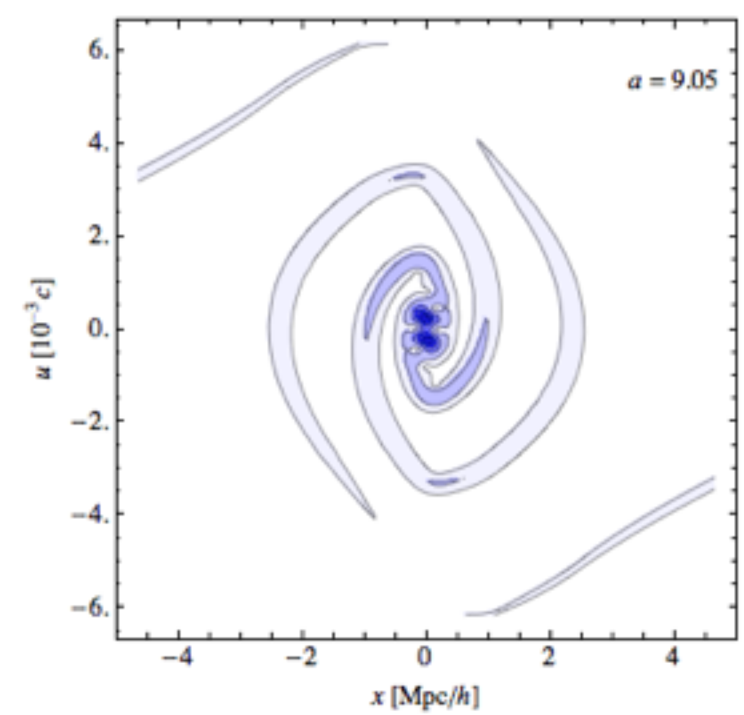
blue S: Schrödinger method
red Z: Zeldovich solution

Virialization

❌ even in extended models:
no virialization

✅ Schrödinger method:
bound structures



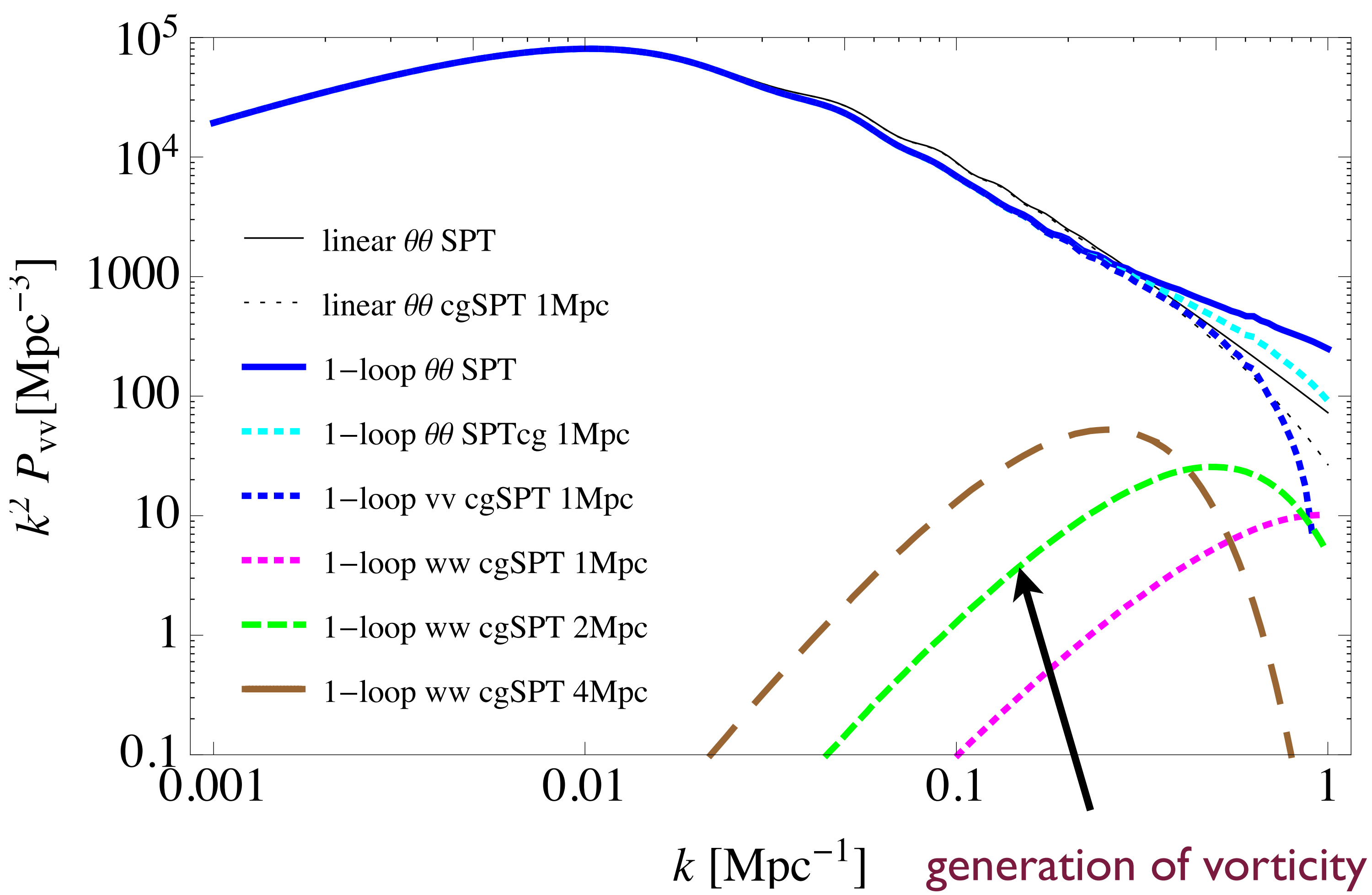


Eulerian Perturbation Theory

Coarse grained dust model (only σ_x)

- similar to dust model, but mass-weighted velocity $\bar{\boldsymbol{v}} := \frac{\overline{n\boldsymbol{v}}}{\bar{n}}$
- velocity divergence $\bar{\theta} := \boldsymbol{\nabla} \cdot \bar{\boldsymbol{v}} \neq \Delta\phi$
- vorticity $\bar{\boldsymbol{w}} := \boldsymbol{\nabla} \times \bar{\boldsymbol{v}} \neq 0$!

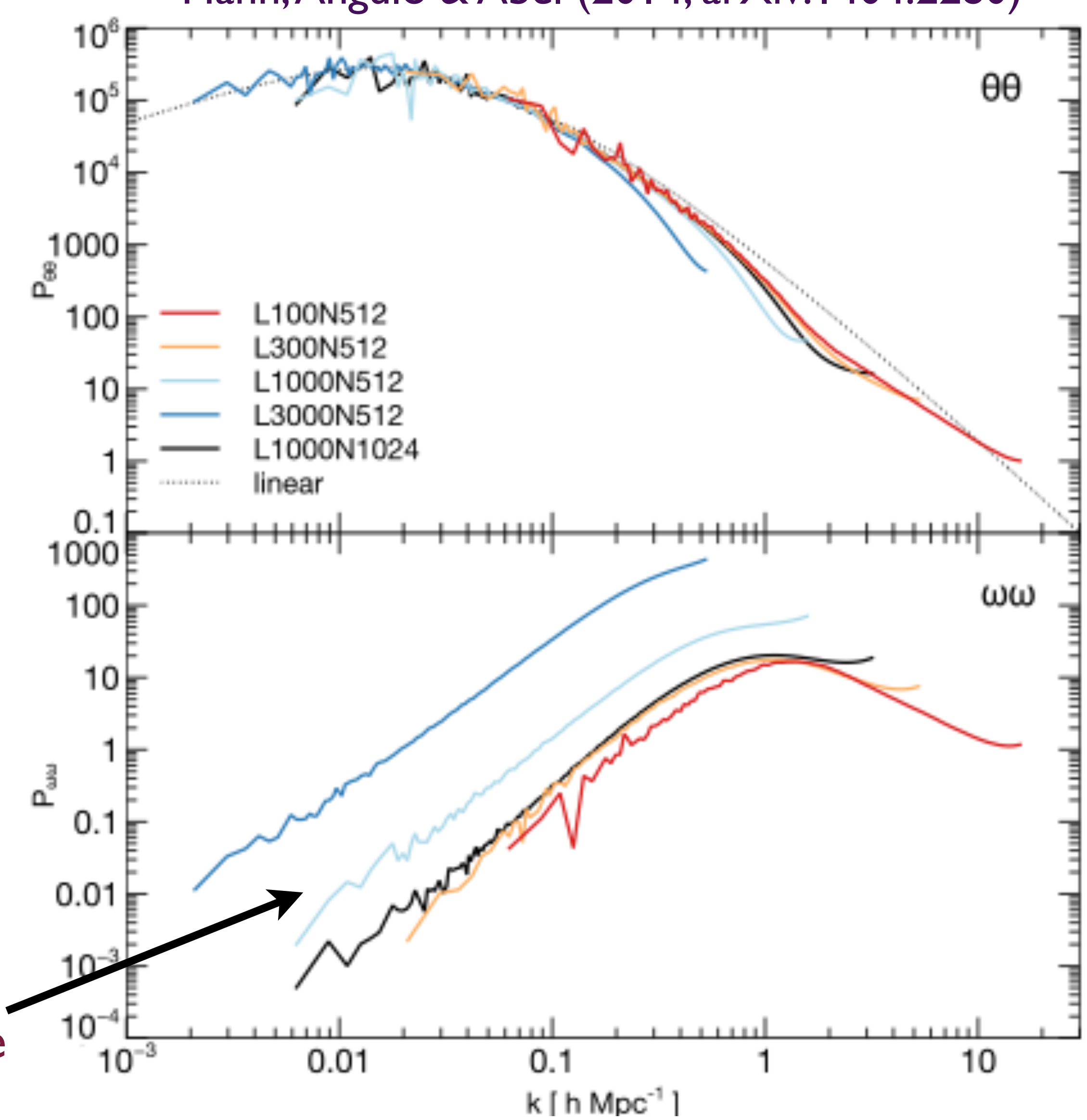
Velocity power spectrum $P_{vv}(k)$: $P_{\theta\theta}(k)$ & $P_{ww}(k)$



generation of vorticity
same form & dependence
as measured in N-body

corresponding N-body data

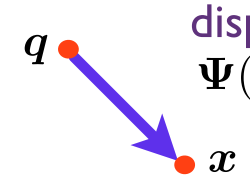
Hahn, Angulo & Abel (2014, arXiv:1404.2280)



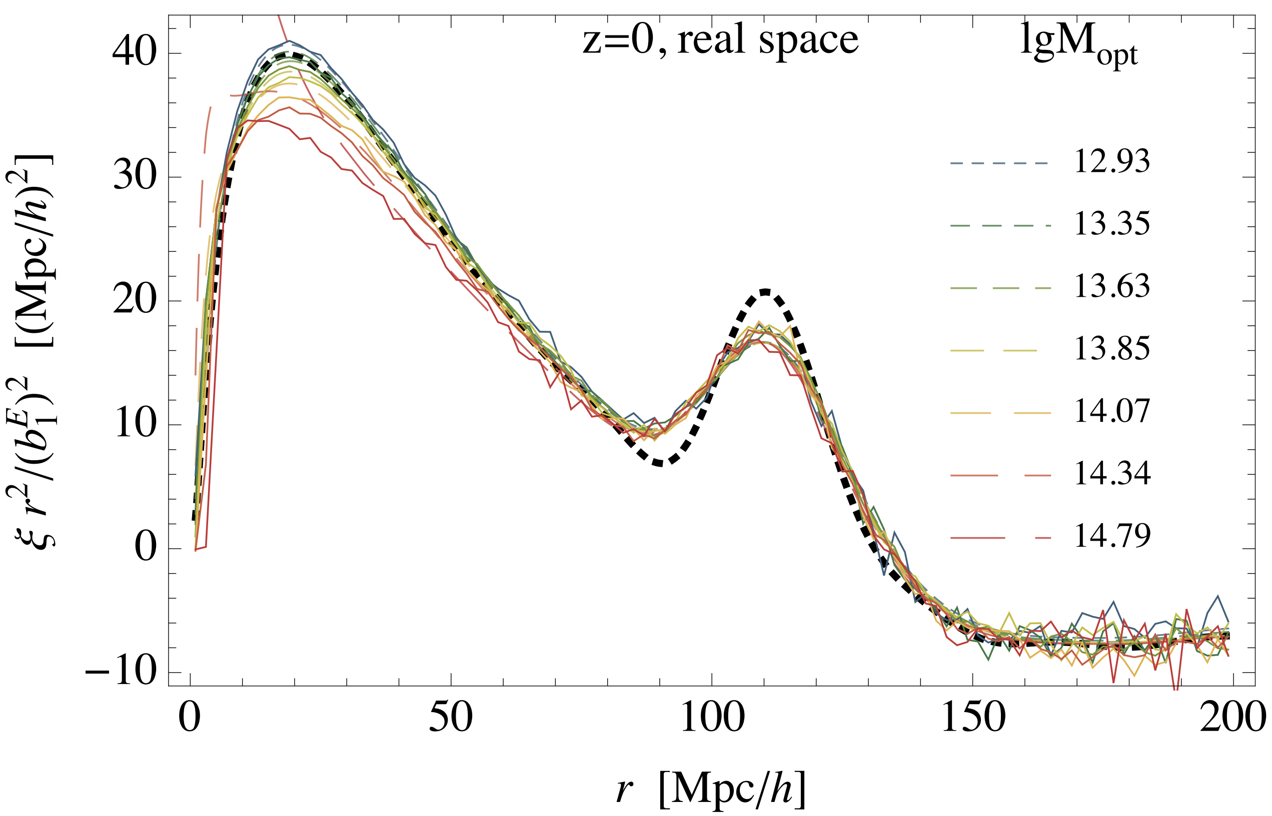
Lagrangian Perturbation Theory

Coarse grained dust model CLPT

- incl. 'truncated Zel'dovich' approximation
- lowest order: smoothed input power spectrum



Carlson et al.
(2012, MNRAS 429)



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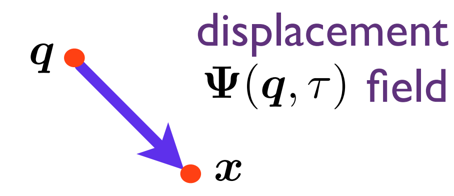

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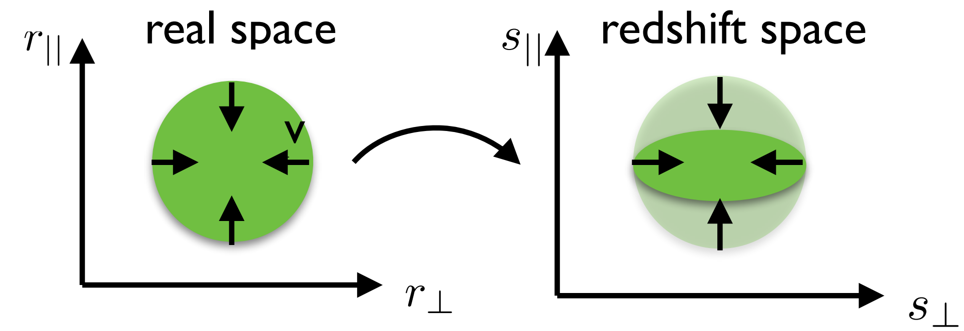
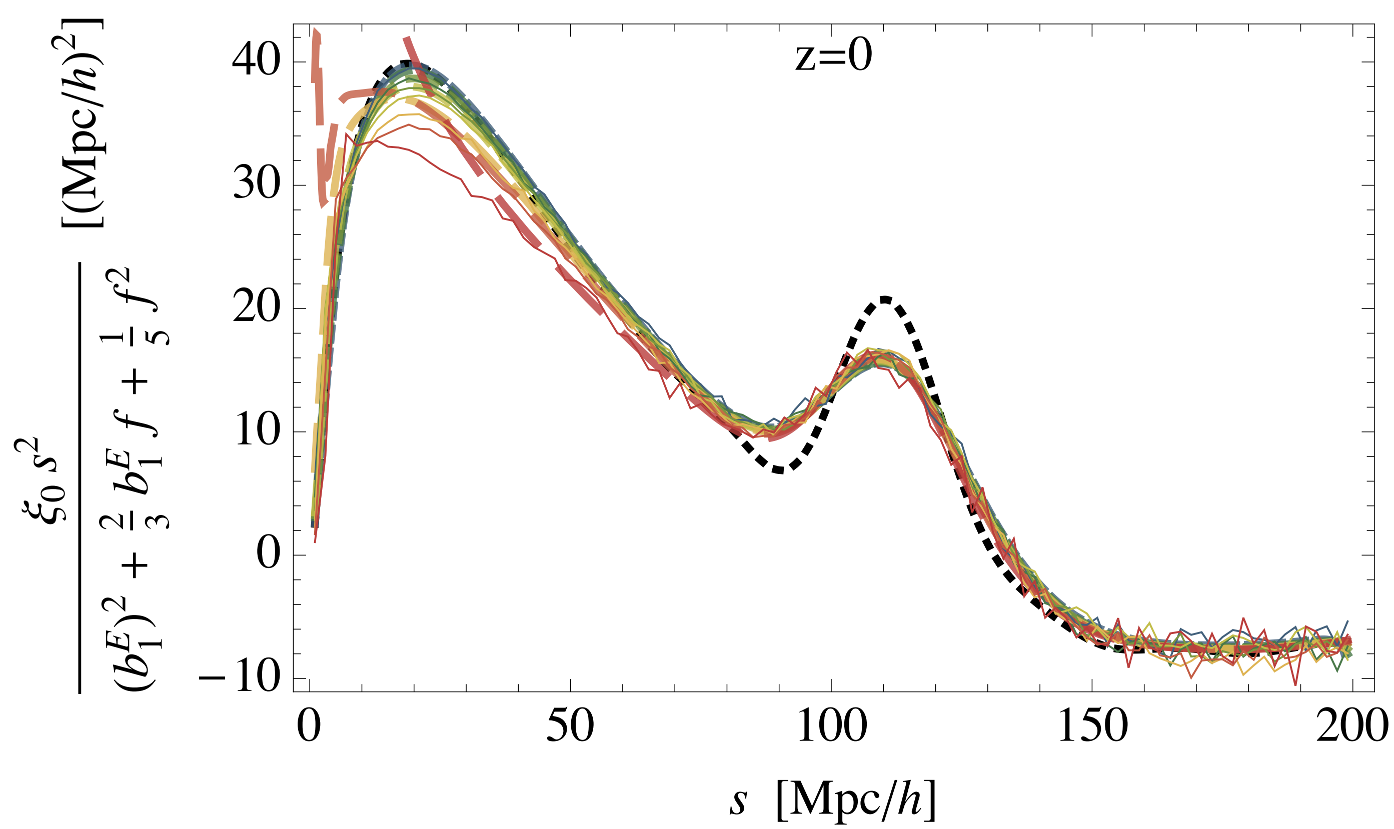
Redshift space distortions: Gaussian streaming model

- generalized to multiple streams & higher orders
- corrections by higher cumulants





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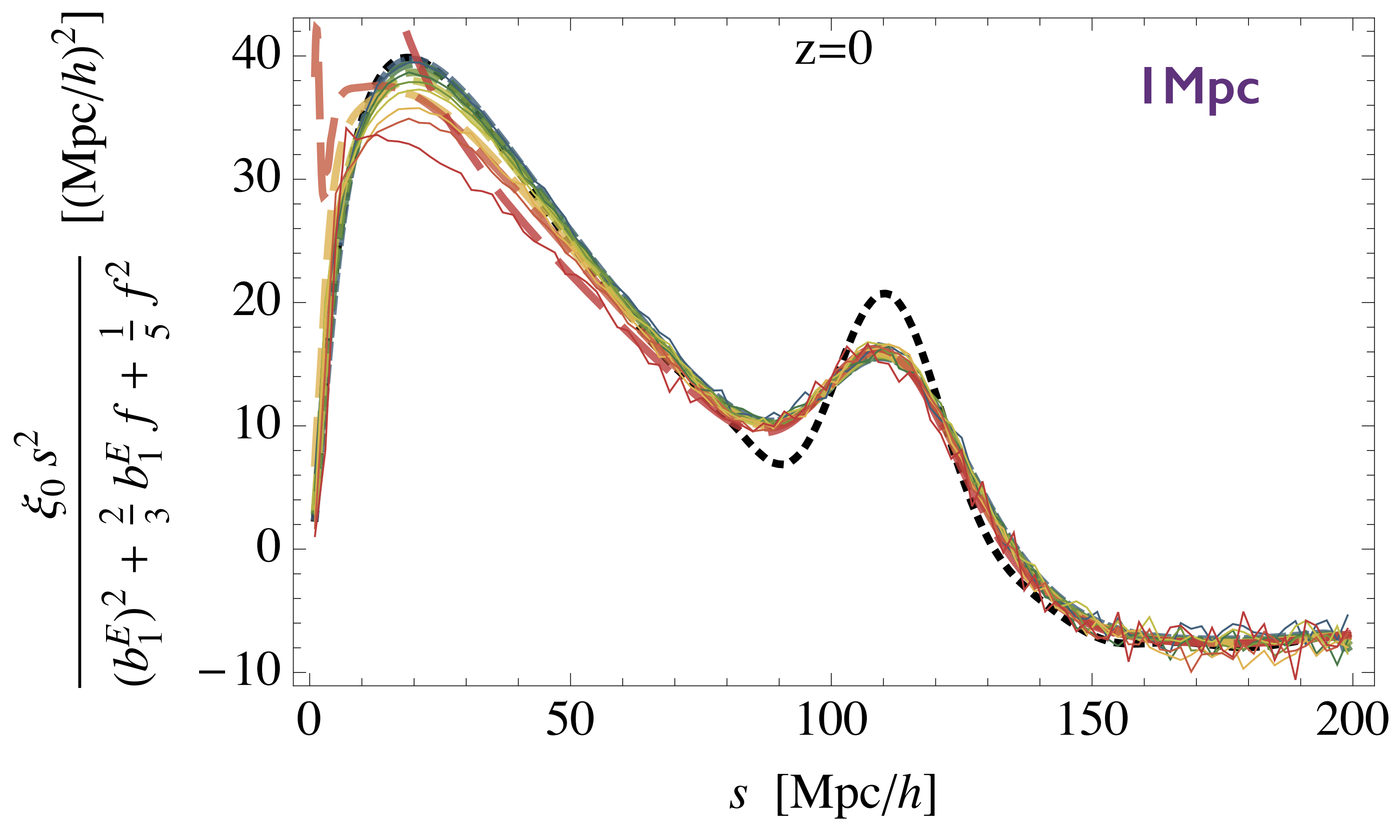
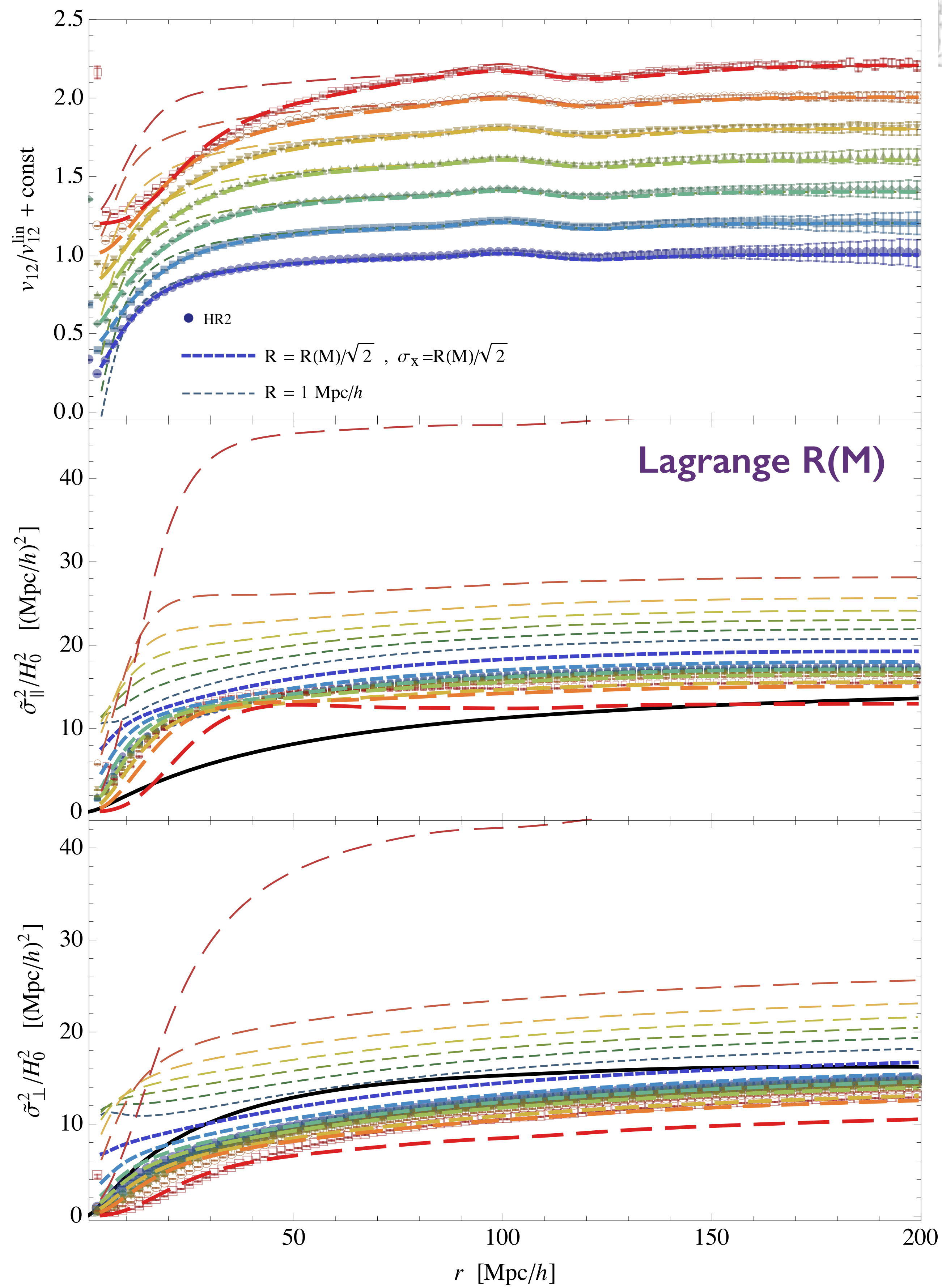
Redshift space distortions: Gaussian streaming model

- generalized to multiple streams & higher orders
- corrections by higher cumulants
- **hybrid model: different smoothing scales** correlation ξ & Gaussian streaming v_{12}, σ_{12}



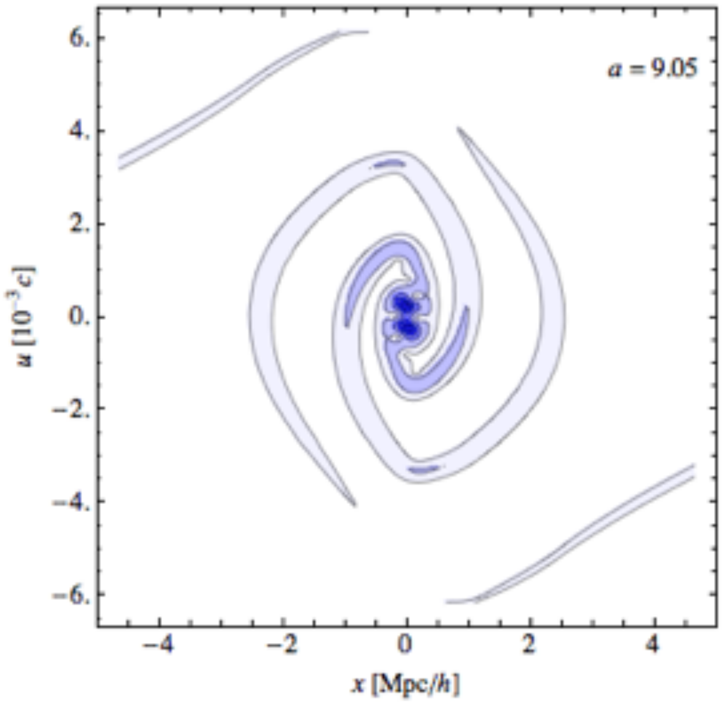


Conclusion & Prospects

Schrödinger method

- models CDM using a self-gravitating scalar field
- analytical tool to access nonlinear stage of structure formation
  - describes multi-streaming
  - allows for virialization

CU, Kopp, Haugg (2014, PRD ??)
arXiv: 1403.5567 [astro-ph.CO]



Coarse-grained dust model in PT

- vorticity from first principles compatible with N-body
- generalization of truncated Zel'dovich approximation

CU, Kopp, Haugg, Achitouv, Weller, Hofmann - to be published soon

Future research

- understand universal density profiles of halos (NFW)
  - search stationary solutions / ground states of gravitational collapse
- employ renormalization group techniques
  - flow of time or phase-space resolution $\hbar$
  - possible interpretation in terms of phase transition
- describe warm dark matter, relativistic & nonrelativistic neutrinos