Search for smooth and non-smooth deviations from the Harrison-Zeldovich spectrum of primordial density perturbations

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The Harrison-Zeldovich spectrum – history

Inflationary predictions for metric perturbations

Outcome of recent CMB observations

Consequences of the would be discovery of primordial GW

Conclusions

The Harrison-Zeldovich spectrum – short history

First step: particle creation and vacuum polarization in an external homogeneous gravitational field

Similar to electron-positron creation by an electric field. From the diagrammatic point of view: one-loop correction to the propagator of an external gravitational field from all quantum matter fields. Particle creation: an imaginary part of this diagram, finite outside singularities, does not require renormalization. Total average value of the EMT: includes real particle creation and vacuum polarization, requires renormalization.

First pioneer papers of Ya. B. Zeldovich on this topic:
1. Ya. B. Zeldovich. Particle production in cosmology. Pisma Zh. Eksp. Teor. Phys. **12**, 443 (1970) [JETP Lett. **12**, 307 (1970)].
Estimate for the energy density of created particles:
$\varepsilon \sim \hbar c^{-3} t^{-4}$.

2. L. P. Pitaevski, Ya. B. Zeldovich. On the possibility of creation of particles by a classical gravitational field. Comm. Math. Phys. **23**, 185 (1971).

To obtain a non-zero $< T_{\mu\nu} >$ from vacuum, the weak or null energy conditions should be violated.

3. Ya. B. Zeldovich, A. A. Starobinsky. Particle creation and vacuum polarization in an anisotropic gravitational field. Zh. Eksp. Teor. Phys. **61**, 2161 (1971) [Sov. Phys. – JETP **34**, 1159 (1972)].

First method of regularization ("n-wave regularization", later proved to be equivalent to the adiabatic regularization proposed by Fulling and Parker in 1974), proof of the Zeldovich 1970 estimate up to the $\ln t$ multiplier in vacuum polarization (not in creation of real particles). Also later proved to produce the correct value for the EMT trace conformal anomaly if first calculated for $m \neq 0$ and then the limit $m \to 0$ is taken.

Second step: add metric fluctuations - requires space-time metric quantization.

Now it becomes a genuine quantum-gravitational effect:

1. Scalar perturbations: acoustic waves (phonons) inside the Hubble radius.

2. Tensor perturbations: graviton creation.
Initially thought to occur near anisotropic singularity. After the Grishchuk (1974) paper, it was understood that similar amount of graviton creation occurs in the isotropic case.

3. No primordial vector perturbations: incompatible with the isotropic behaviour of the early Universe.

Metric fluctuations and the Harrison-Zeldovich initial spectrum

Third step: from particles to field and metric fluctuations.

For scales exceeding the Hubble radius H^{-1} where $H \equiv \frac{\dot{a}(t)}{a(t)}$, the particle notion is not well defined. Instead, the field fluctuations themselves become observable.

However, for a power-law behaviour of space-time metric coefficients, scalar metric fluctuations generated from a vacuum initial state have a strongly blue-tilted spectrum and are very small at astronomical scales. So, after an unsuccessful attempt to use phonon creation to produce a flat (scale-free) initial spectrum of scalar perturbations (1972), Zeldovich introduced it by hand:
Ya. B. Zeldovich. A hypothesis, unifying the structure and the entropy of the Universe. Mon. Not. R. Astr. Soc. **160**, L1 (1972).

Mon. Not. R. astr. Soc. (1972) **160**, *Short Communication.*

A HYPOTHESIS, UNIFYING THE STRUCTURE AND THE ENTROPY OF THE UNIVERSE

Ya. B. Zeldovich

(Received 1972 September 4)

SUMMARY

A hypothesis about the averaged initial state and its perturbations is put forward, describing the entropy of the hot Universe (due to damping of short waves) and its structure (clusters of galaxies due to long wave perturbations).

A hypothesis is put forward, assuming that initially, near the cosmological singularity, the Universe was filled with cold baryons. The averaged evolution was described by the uniform isotropic expansion, according to Friedmann solution and the equation of state of cold baryons.

Superimposed on this averaged picture are initial fluctuations of baryon density and corresponding fluctuations of the metric.

One unique value (approximately 10^{-4}) of non-dimensional amplitude of metric fluctuations, scale-independent, describes two different, most important properties of the Universe—its structure and its entropy. The density fluctuations are in inverse proportion to the square of the scale at a given moment of time. The fluctuations of small scale of the order of mean distance between two neighbouring baryons first increase, soon they are transformed into acoustical waves, i.e. phonons, propagating in the baryonic fluid.

The damping of short acoustic waves is accompanied by their transformation into various modes of excitation and relaxation to thermodynamic equilibrium with high specific entropy per baryon. This line of reasoning with one parameter adjusted (the initial metric perturbation 10^{-4}) leads to the ratio of photons to baryons characteristic for the hot Universe.

The relaxation into thermodynamic equilibrium occurs early, at $t \ll \hbar/m_r c^2$. Therefore the well-known scenario of hot Universe evolution is conserved, including the hadronic era with plenty of antibaryons, the nucleogenesis leading to 25–30 per cent He^4, the radiation dominated era giving blackbody 2·7 K radiation.

Particularly the hadronic era with the ratio of baryons to antibaryons $B : \bar{B} = 1 + 10^{-8}$ seemed to be most unnatural in the standard scenario of the hot Universe. Why is the ratio not 1? Why is the departure from unity (10^{-8}) everywhere positive?

In our hypothesis initially $B : \bar{B} = 1 : 0$ everywhere. Thermal excitations add pairs, $B = \bar{B}$, whose number is 10^8 times greater than the number of initial baryons. Therefore the puzzling $1 + 10^{-8}$ ratio is explained by harmless (although arbitrary) density and metric fluctuations.

The second line of reasoning concerns long wave fluctuations of metric and density.

The characteristic time when $\lambda = c, t$ for these perturbations occurs very late

Remarkable hypothesis which had been producing a very good approximation to super-Hubble scalar metric perturbations for many years. But now we have to go further and look for small deviations from this spectrum.

The situation with initial conditions had changed completely after it was found (first, for gravitons in Starobinsky, 1979) that initial vacuum quantum fluctuations can produce an approximately flat spectrum of perturbations if a quasi-de Sitter stage, $|\dot{H}| \ll H^2$ (later dubbed inflation) preceded the radiation dominated stage in the early Universe (this was first conjectured by Gliner in 1970).

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

Main advantages of inflation

Aesthetic elegance

Inflation – hypothesis about an almost maximally symmetric (quasi-de Sitter) stage of the evolution of our Universe in the past, before the hot Big Bang. If so, preferred initial conditions for (quantum) inhomogeneities with sufficiently short wavelengths exist – the adiabatic in-vacuum ones. In addition, these initial conditions represent an attractor for a much larger compact open set of initial conditions having a non-zero measure in the space of all initial conditions.

Predictability, proof and/or falsification

Given equations, this gives a possibility to calculate all subsequent evolution of the Universe up to the present time and even further to the future. Thus, any concrete inflationary model can be proved or disproved by observational data.

Naturalness of the hypothesis

Remarkable qualitative similarity between primordial and present dark energy.

Present status of inflation

From "proving" inflation to using it as a tool

Present status of inflation: transition from "proving" it in general and testing some of its simplest models to applying the inflationary paradigm to investigate particle physics at super-high energies and the actual history of the Universe in the remote past using real observational data on $n_s(k) - 1$ and $r(k)$.

Evolution of generated metric perturbations

One spatial Fourier mode $\propto e^{i\mathbf{kr}}$ is considered.

For scales of astronomical and cosmological interest, the effect of creation of metric perturbations occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the radiation or matter dominated stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.

Classical-to-quantum transition for the leading modes of perturbations

In the superhorizon regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

Quantum-to-classical transition: in fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta,\ g$).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

FLRW dynamics with a scalar field

In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Inflationary slow-roll dynamics

Slow-roll occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|$, $\dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V$, $|V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \quad \dot{\phi} \approx -\frac{V'}{3H}, \quad N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot\phi^2} = \frac{GH_k^4}{\pi|\dot H|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln\frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d\ln P_\zeta(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

$$|n_s(k) - 1| \ll 1$$

Generically $n_s \neq 1$ – deviation from the Harrison-Zeldovich spectrum is expected!

The special case when $n_s \equiv 1$: $V(\phi) \propto \phi^{-2}$ in the slow-roll approximation.

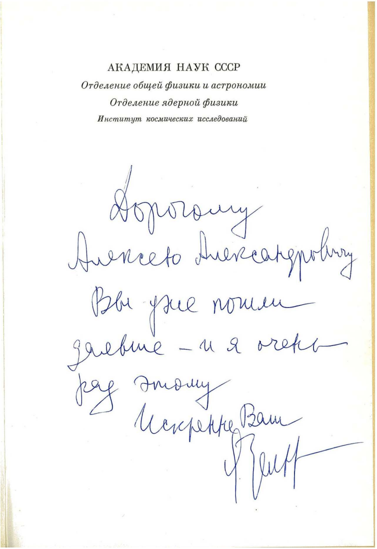
Omitting the slow-roll assumption:

let $x = \sqrt{4\pi G}\phi, \quad y = B\sqrt{4\pi G}H, \quad v(x) = \frac{32\pi^2 G^2 B^2}{3} V(\phi)$.

Then (A. A. Starobinsky, JETP Lett. 82, 169 (2005)):

$$y = e^{x^2/2}\left(\int_x^\infty e^{-\tilde{x}^2/2}\, d\tilde{x} + C\right)$$

$$v = y^2 - \frac{1}{3}\left(\frac{dy}{dx}\right)^2$$



АКАДЕМИЯ НАУК СССР

Отделение общей физики и астрономии

Отделение ядерной физики

Институт космических исследований

Дорогому
Алексею Александровичу
Вы уже пошли
дальше — и я очень
рад этому
Искренне Ваш

Translation "To dear Alexei Alexandrovich
You have stepped further already – and I am very happy with this   Yours sincerely   Ya. Zeldovich"

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50-60)$.

Two kinds of deviations

1. Smooth deviations.
Small and slowly varying $n_s(k) - 1$ and $r(k)$.
Expected.

2. Non-smooth deviations.
Require temporal break of slow-roll conditions. This in turn requires some new physics during inflation.
May be but need not be.

Outcome of recent CMB observations

I. A year ago

The most important for the history of the early Universe are:

1. The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$P_\zeta(k) = \int \frac{\Delta_\zeta^2(k)}{k} dk, \quad \Delta_\zeta^2 = \left(2.20^{+0.05}_{-0.06}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.040 \pm 0.007$$

N.B.: The value is obtained under some natural assumptions, the most critical of them is $N_\nu = 3$, for $N_\nu = 4$ many things have to be reconsidered and $n_s \approx 1$ is not excluded.

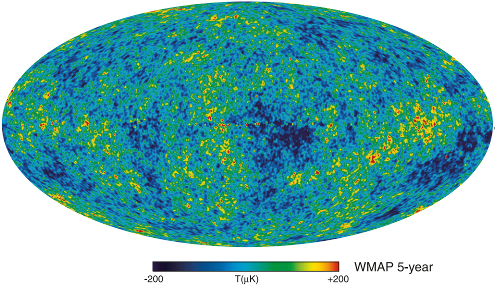
2. Neither the B-mode of CMB polarization, nor primordial GW were discovered: $r < 0.11$ at the 95% CL.

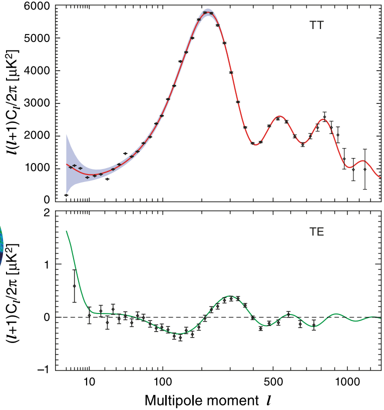
NB: The assumption: $n_s - 1 = -\frac{2}{N} \approx -0.04$ for all $N = 1 - 60$ implies a lower bound on r. In particular, if $r \ll 8|n_s - 1|$, then

$$V(\phi) = V_0\left(1 - \exp(-\alpha\kappa\phi)\right)$$

with $\alpha\kappa\phi \gg 1$ but α not very small, and

$$r = \frac{8}{\alpha^2 N^2}$$


WMAP 5-year
-200
T(μK)
+200


TT
TE
l(l+1)C_l/2π [μK²]
(l+1)C_l/2π [μK²]
6000
5000
4000
3000
2000
1000
0
2
1
0
–1
10
100
500
1000
Multipole moment l

Combined results from Planck and other experiments

P. A. R. Ade et al., arXiv:1303.5082

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	$0.9643 + 0.0059$
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002$ Mpc^{-1}.

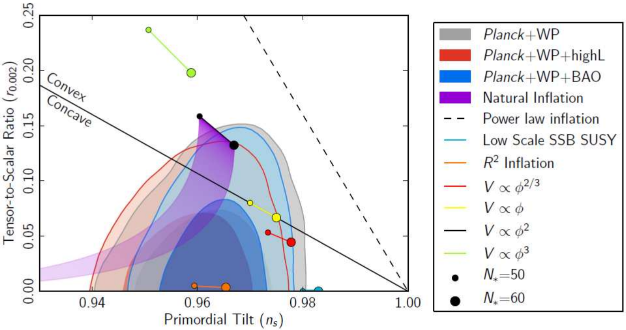


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

II. Three months ago

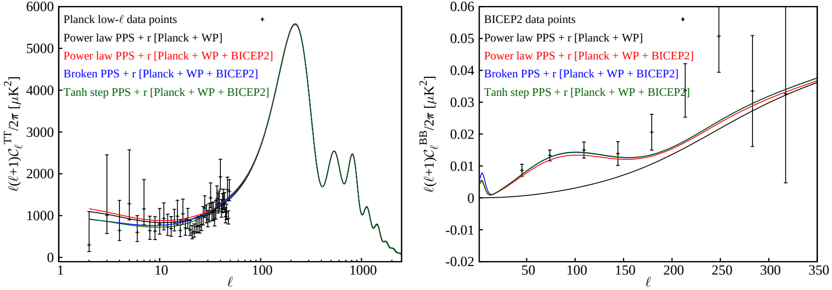
The BISEP2 collaboration announced the discovery of the B-mode in the multipole range $30 < \ell < 150$: P. A. R. Ade et al., arXiv:1403.3985 (for larger l it was discovered earlier this year with the amount in agreement from gravitational lensing of scalar perturbations) with

$$r = 0.20^{+0.07}_{-0.05}$$

The unsubtracted result – contains an unknown foreground non-thermal part.

Consequence – assuming the Einstein gravity:

$$\sqrt{G} H_{dS} = 0.99 \times 10^{-5} \left(\frac{r_{0.002}}{0.2} \right)^{1/2} 5^{0.96 - n_s}$$


Planck low-ℓ data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{TT}/2\pi\ [\mu K^2]$
ℓ
BICEP2 data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)C_\ell^{BB}/2\pi\ [\mu K^2]$
ℓ

Consequences of the would be discovery of primordial GW

If confirmed by an independent measurement:

1. Discovery of a real singularity – a state of the Universe in the past with a very high curvature (with H only 5 orders of magnitude less than the Planck mass).

2. Discovery of a new class of gravitational waves – primordial ones.

3. Decisive argument for the necessity of quantization of gravitational waves.

4. Decisive test of the inflationary paradigm as a whole.

5. Discovery of $\sim 20\%$ deviation of the power spectrum of scalar perturbations from a scale-free one – new physics during inflation!

Non-smooth feature in the power spectrum

The most intriguing discordance between WMAP and Planck results from one side and the BISEP2 ones from the other: no sign of GW in the CMB temperature anisotropy spectrum.

Instead of the $1 + \frac{6.20}{8} r \approx 1.1$ increase of the total anisotropy power spectrum over the multipole range $2 \ll \ell < 50$, a $\sim 10\%$ depression is seen for $20 \lesssim \ell \lesssim 40$ (see e.g. Fig. 39 of arXiv:1303.5076).

The feature exists even if $r \ll N^{-1}$ but the presence of $r \sim 0.1$ makes it larger.

6.20 is the rounded value for

$$\frac{25}{9}\left(1+\frac{48\pi^2}{385}\right)$$

(A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985)). In this expression, only the Sachs-Wolfe effect is taken into account for scalar perturbations. Adding the Doppler and Silk effects leads to the decrease of this number to ≈ 5 for the range $20 \lesssim \ell \lesssim 40$.

More detailed analysis in D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1403.7786 : the power-law form of $P_\zeta(k)$ is excluded at more than 3σ CL.

Broken scale models describing both WMAP-Planck and BISEP2 data

Next step: "whipped inflation" D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1404.0360.

The model contains a new scale at which the effective inflaton potential has a feature which the inflaton crosses about 50 e-folds before the end of inflation. The existence of such a feature, in turn, requires some new physics (e.g. fast phase transition in a second field coupled to the inflaton).

$$V(\phi) = V_S(\phi) + V_R(\phi)$$

$$V_S(\phi) = \gamma\phi^p, \quad V_R(\phi) = \lambda(\phi - \phi_0)^q \Theta(\phi - \phi_0)$$

Best results for $(p, q) = (2, 3)$.

Wiggles in the power spectrum

The effect of the same order: an upward wiggle at $\ell \approx 40$ and a downward one at $\ell \approx 22$.

Lesson: irrespective of a future analysis of foreground contamination in the BISEP2 result, features in the anisotropy spectrum for $20 \lesssim \ell \lesssim 40$ confirmed by WMAP and Planck should be taken into account and studied seriously.

A more elaborated class of model suggested by previous studies of sharp features in the inflaton potential caused, e.g. by a fast phase transition occurred in another field coupled to the inflaton during inflation:
WWI (Wiggly Whipped inflation)
D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, arXiv:1405.2012

In particular, the potential with a sudden change of its first derivative:

$$V(\phi) = \gamma\phi^2 + \lambda\phi^p(\phi - \phi_0)\,\theta(\phi - \phi_0)$$

which generalizes the exactly soluble model considered in A. A. Starobinsky, JETP Lett. **55**, 489 (1992) produces $-2\Delta\ln\mathcal{L} = -11.8$ compared to the best-fitted power law scalar spectrum, partly due to the better description of wiggles at both $\ell \approx 40$ and $\ell \approx 22$.

A sharp feature in the potential leads to a rapid increase of the effective inflaton mass, $m^2 = V''(\phi)$, in the vicinity of $\phi = \phi_0$. While $m \approx 2 \times 10^{13}$ GeV for $\phi < \phi_0$, it becomes of the order of 10^{14} GeV and larger at earlier times when $\phi \geq \phi_0$ (but still much less than the energy density scale of the inflaton potential $\sim 3 \times 10^{16}$ GeV).

Conclusions

- On one hand, the measured primordial scalar (adiabatic) power spectrum is not far from the Harrison-Zeldovich one: $|n_s - 1| \ll 1$. On the other hand, its smooth deviation from the HZ spectrum is discovered definitely.
- First, though inconclusive, evidence for primordial GW generated during inflation. The BISEP2 result taken literally is the confirmation of the general prediction (made in 1979) of the early Universe scenario with the de Sitter (inflationary) stage preceding the radiation dominated stage (the hot Big Bang).
- The derivation of this prediction is based on gravity (space-time metric) quantization and requires very large space-time curvature in the past of our Universe with a characteristic length only five orders of magnitude larger than the Planck one.

- However, would the BISEP2 result be confirmed, inflation is not so simple: the scalar primordial power spectrum deviates from a scale-free one that implies the existence of some scale (i.e. new physics) during inflation.
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.
- The conceptual change in utilizing CMB and other observational data from "proving inflation" to using them to determine the spectrum of particle masses in the energy range $(10^{13} - 10^{14})$ GeV by making a "tomographic" study of inflation.
- Ya. B. Zeldovich was a great man, made much and was at the right track in this area of cosmology, but we have to go further!