Quantifying the Cosmic Web using Shapefinder diagonistic

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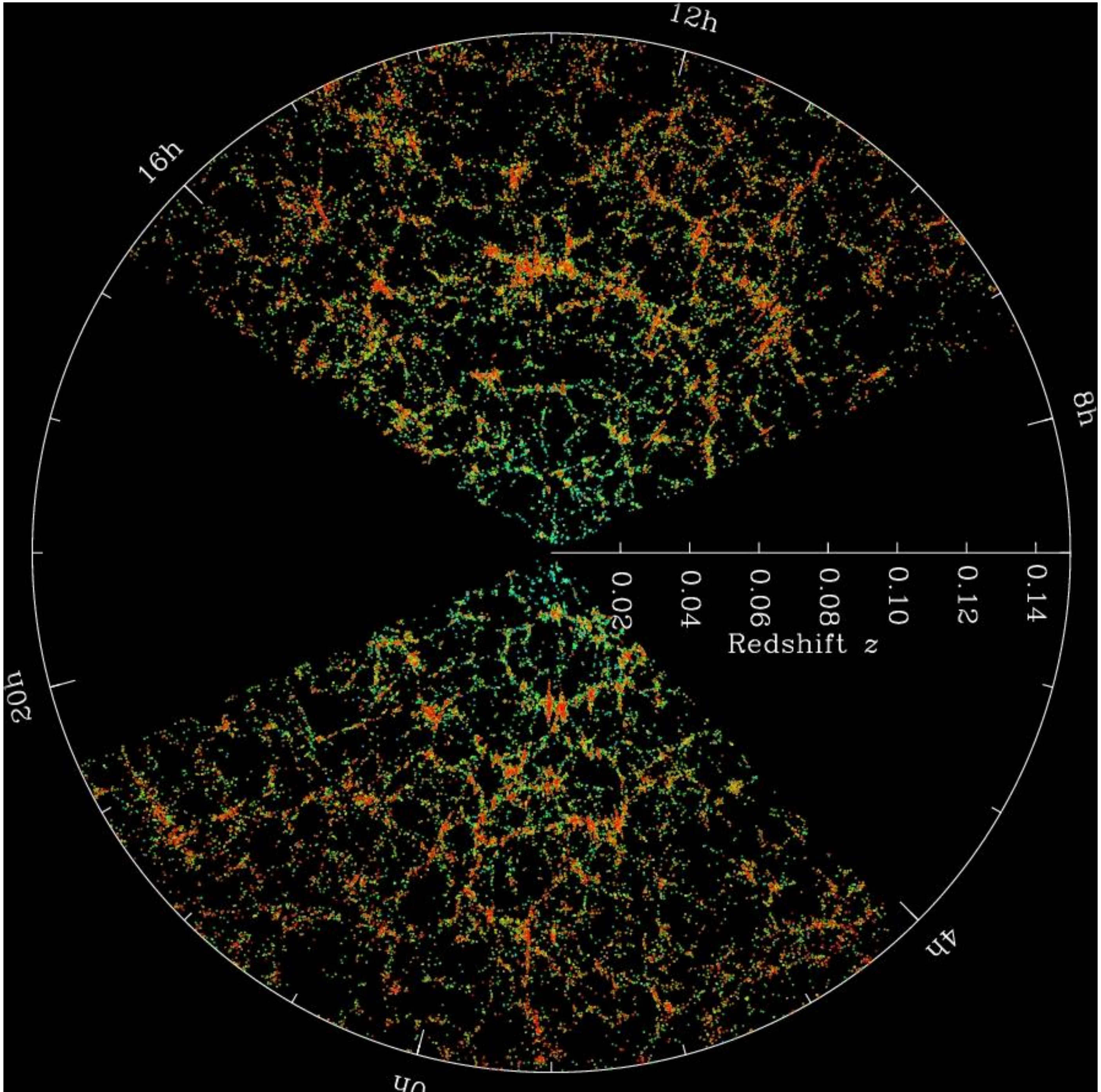
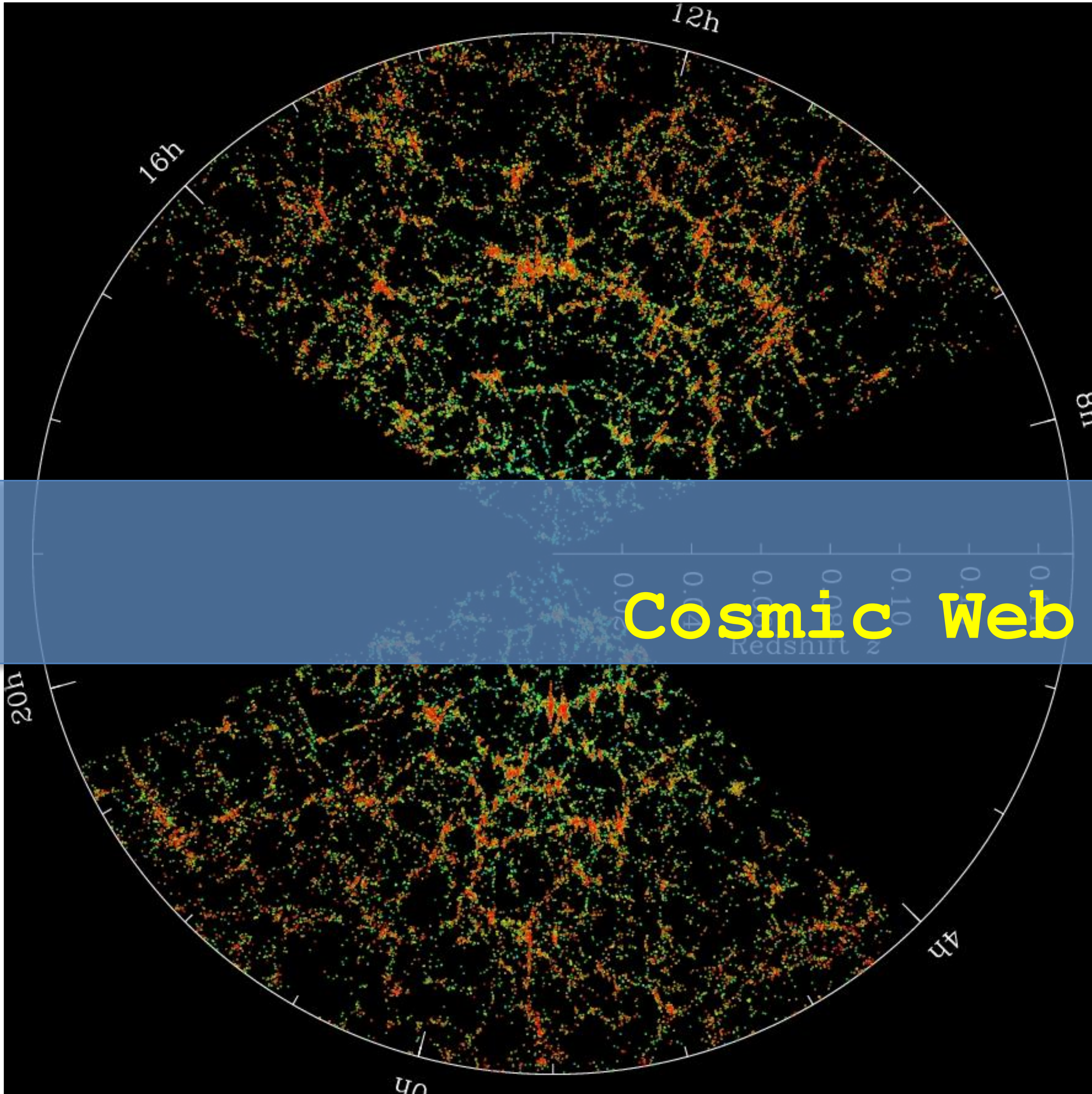


Credit: M. Blanton and the Sloan Digital Sky Survey.

Cosmic Web



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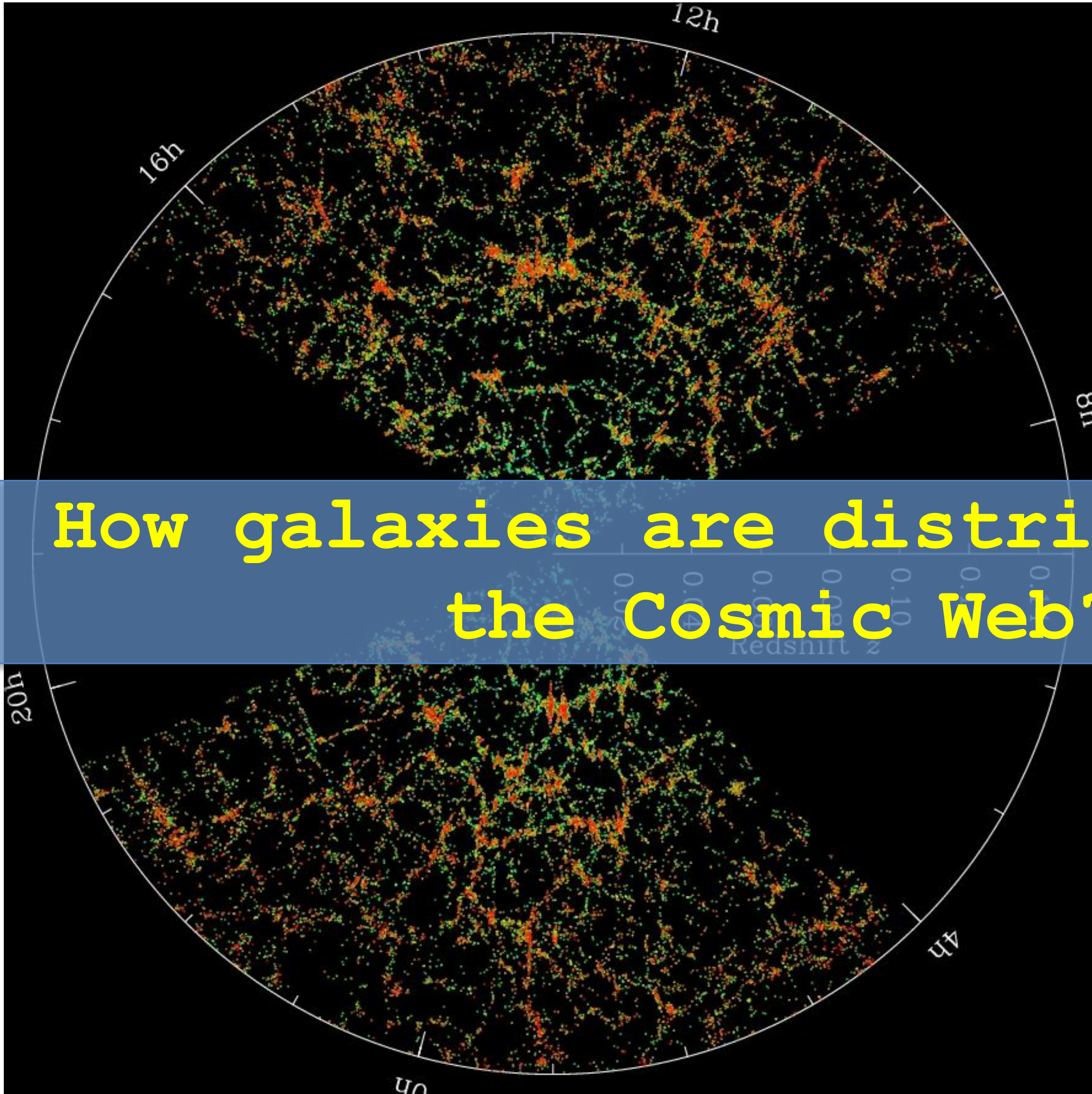
How galaxies are distributed in the Cosmic Web?



Credit: M. Blanton and the Sloan Digital Sky Survey.

Shapefinder diagonistic

In 3D the four Minkowski Functionals are:

1. Volume (V)
2. Surface Area (S)
3. Integrated Mean Curvature (C)
4. Euler Characteristics (χ) or Genus (G)

$$C = \int\int \frac{\kappa_1 + \kappa_2}{2} dS,$$

Shape parameters having dimension of Length

$$\mathcal{T} = \frac{3V}{S}, \quad \mathcal{B} = \frac{S}{C}, \quad \mathcal{L} = \frac{C}{4\pi G}.$$

Dimensionless Shapefinders – Filamentarity $(\mathcal{F})$ and Planarity $(\mathcal{P})$

$$\mathcal{P} = \frac{\mathcal{B} - \mathcal{T}}{\mathcal{B} + \mathcal{T}}, \mathcal{F} = \frac{\mathcal{L} - \mathcal{B}}{\mathcal{L} + \mathcal{B}}$$

Solid Sphere (R): $V = \frac{4}{3}\pi R^3$, $S = 4\pi R^2$, $C = 4\pi R$, $\chi = 2$

$$\Rightarrow \mathcal{L} = \mathcal{B} = \mathcal{T} = R, \ \mathcal{F} = \mathcal{P} = 0$$

Shapefinder diagonistic

- Minkowski Functionals and shapefinders are defined on surface.
- But observational data shows point distribution of galaxy.

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How can we construct a surface from point distribution?

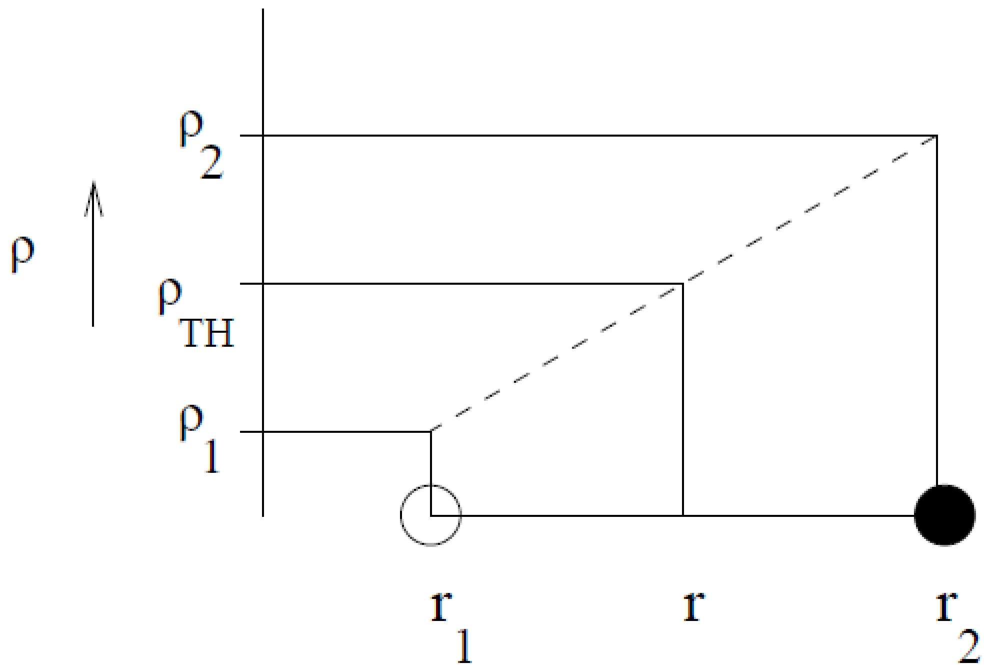
SURFace GENerator (SURFGEN)

SURFGEN

- Written in C language.
- Generate surface whose physical origin is quite varied and different.
- Input : density field defined on a rectangular grid .
- Using Marching cube 33 algorithm it generates the isodensity surface.

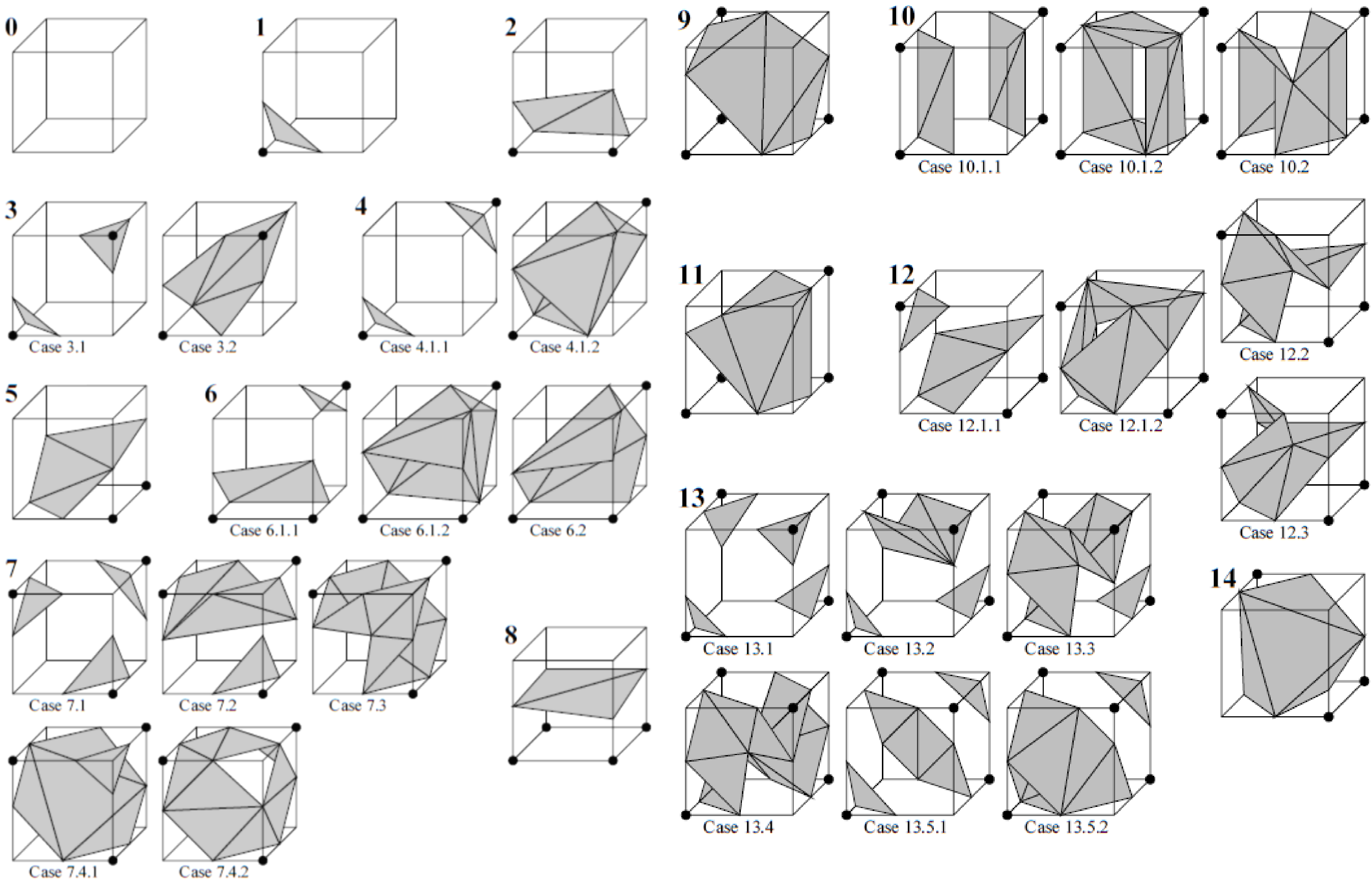
Marching cube

- Generate triangulated surface from a 3D density field defined on a rectangular grid.
- It determines the polygon(s) needed to represent the part of the isosurface ($\rho=\rho_{th}$) that passes through this cube. The individual polygons are then fused into the desired surface.

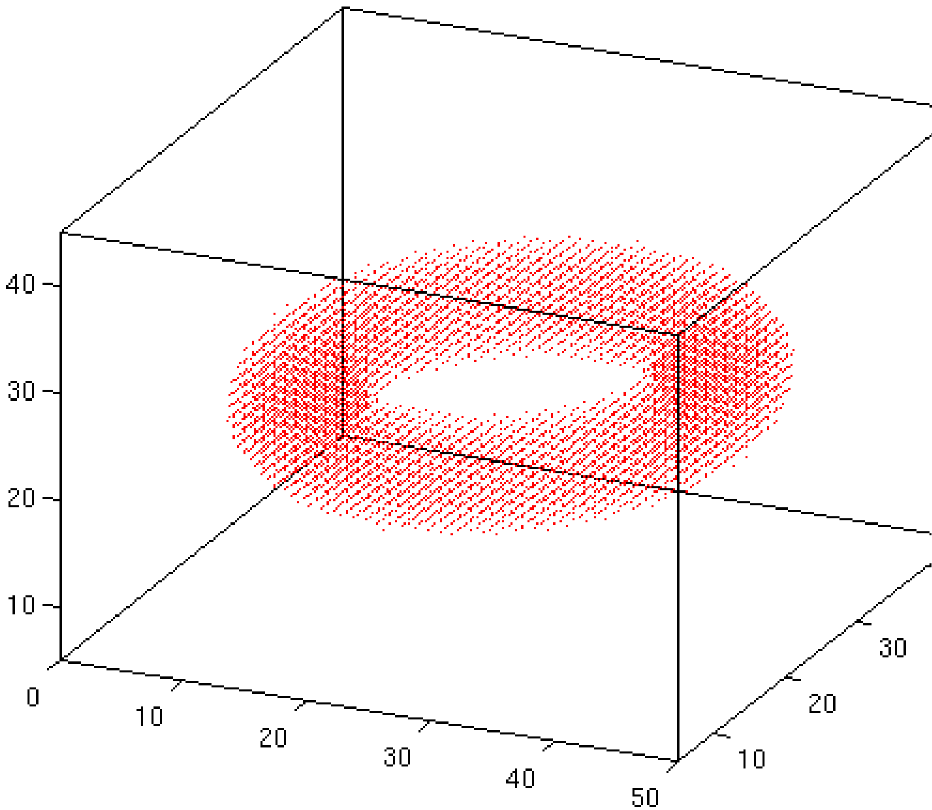


$$\mathbf{r} = \mathbf{r}_2 - \frac{\rho(\mathbf{r}_2) - \rho_{\mathrm{TH}}}{\rho(\mathbf{r}_2) - \rho(\mathbf{r}_1)} \times (\mathbf{r}_2 - \mathbf{r}_1),$$

Jatush's thesis



Marching cube - 33 lookup table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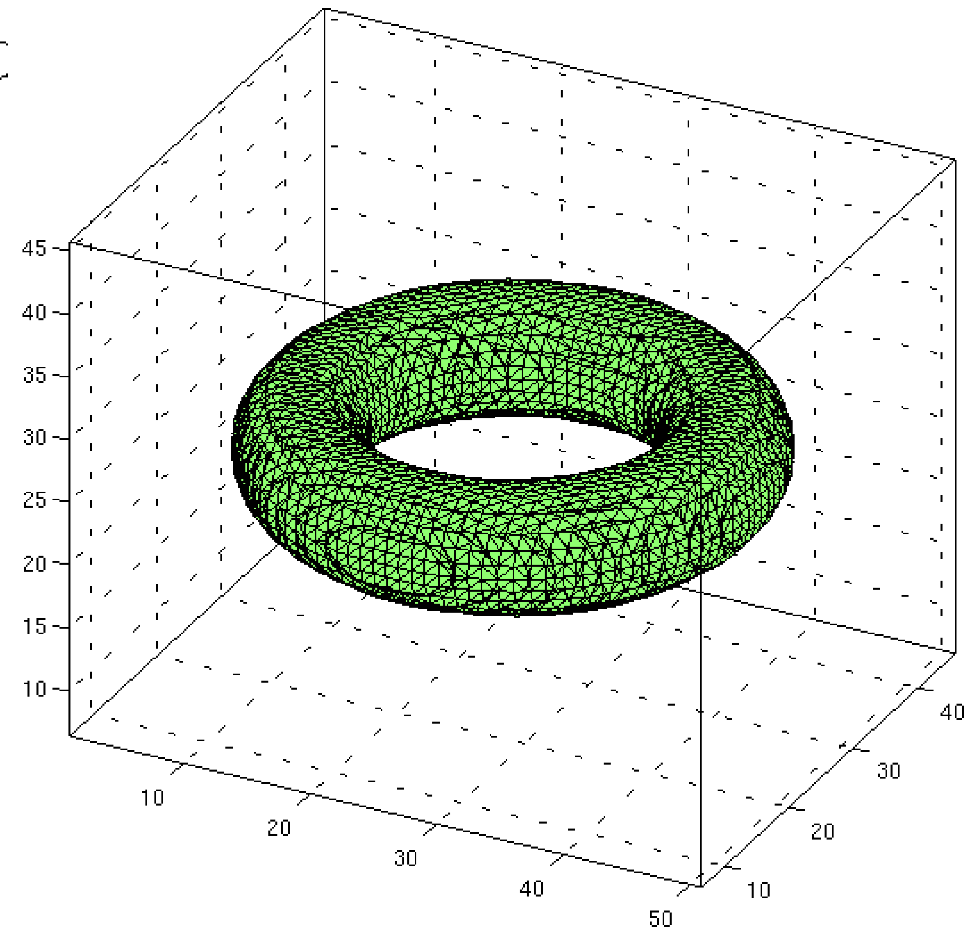
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Minkowski Functionals

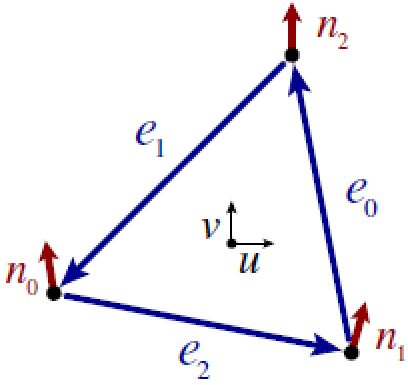
$$S = \sum_{i=1}^{N_T} S_i,$$

$$V = \sum_{i=1}^{N_T} V_i, \quad V_i = \frac{1}{3} S_i \{n_j P^j\}_i,$$

$$\chi = N_T - N_E + N_V$$

Integrated Mean Curvature is calculated using per-vertex method describe in Rusinkiewicz (2004).

II : Weingarten matrix



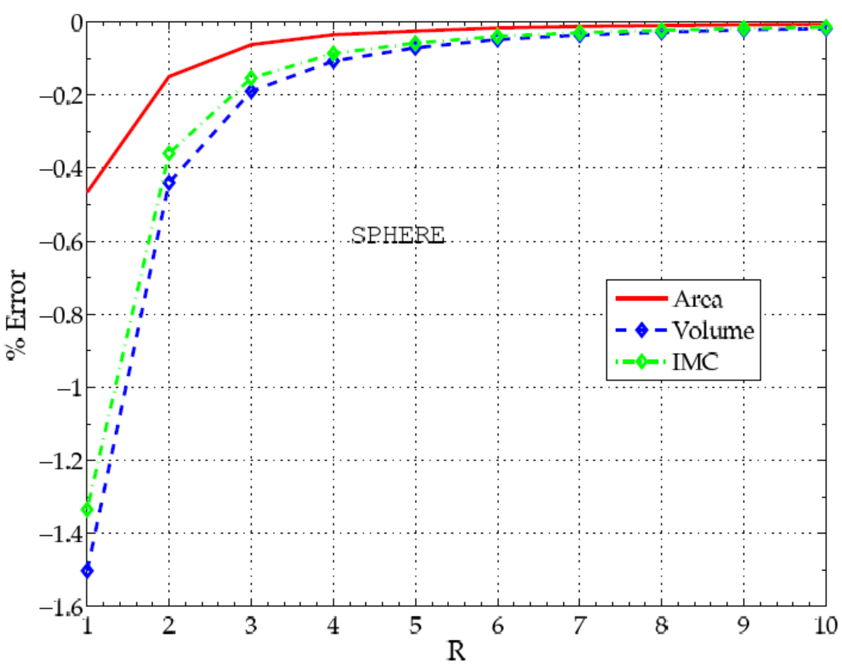
$$\mathbf{II}\begin{pmatrix} e_0 \cdot u \\ e_0 \cdot v \end{pmatrix} = \begin{pmatrix} (n_2 - n_1) \cdot u \\ (n_2 - n_1) \cdot v \end{pmatrix}$$

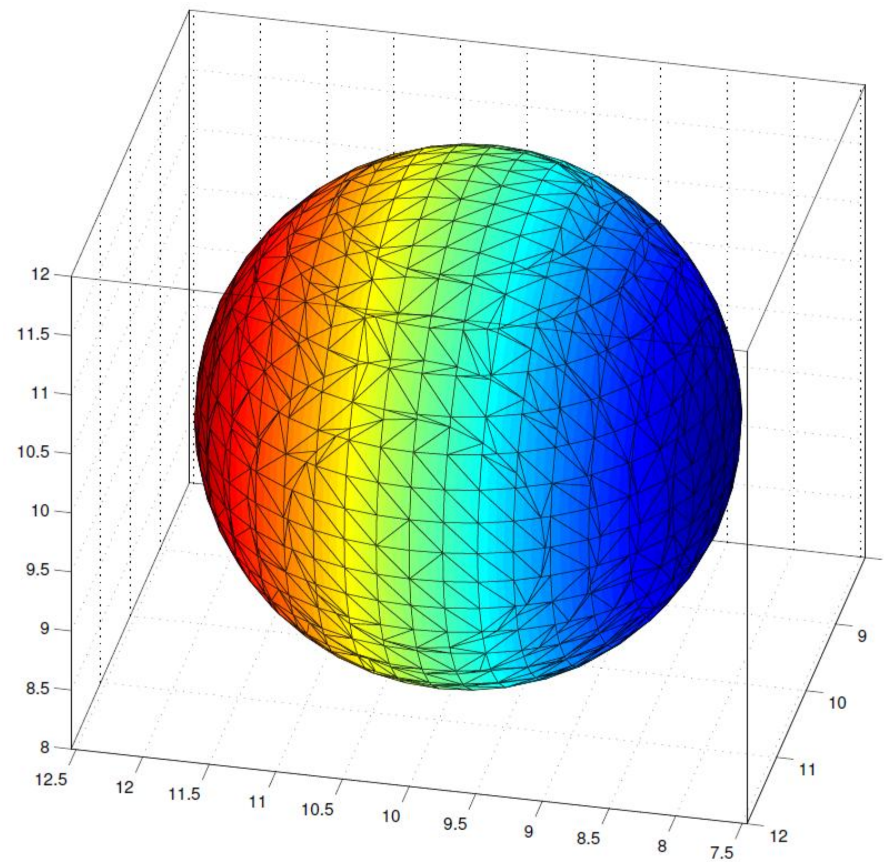
$$\mathbf{II}\begin{pmatrix} e_1 \cdot u \\ e_1 \cdot v \end{pmatrix} = \begin{pmatrix} (n_0 - n_2) \cdot u \\ (n_0 - n_2) \cdot v \end{pmatrix}$$

$$\mathbf{II}\begin{pmatrix} e_2 \cdot u \\ e_2 \cdot v \end{pmatrix} = \begin{pmatrix} (n_1 - n_0) \cdot u \\ (n_1 - n_0) \cdot v \end{pmatrix}$$

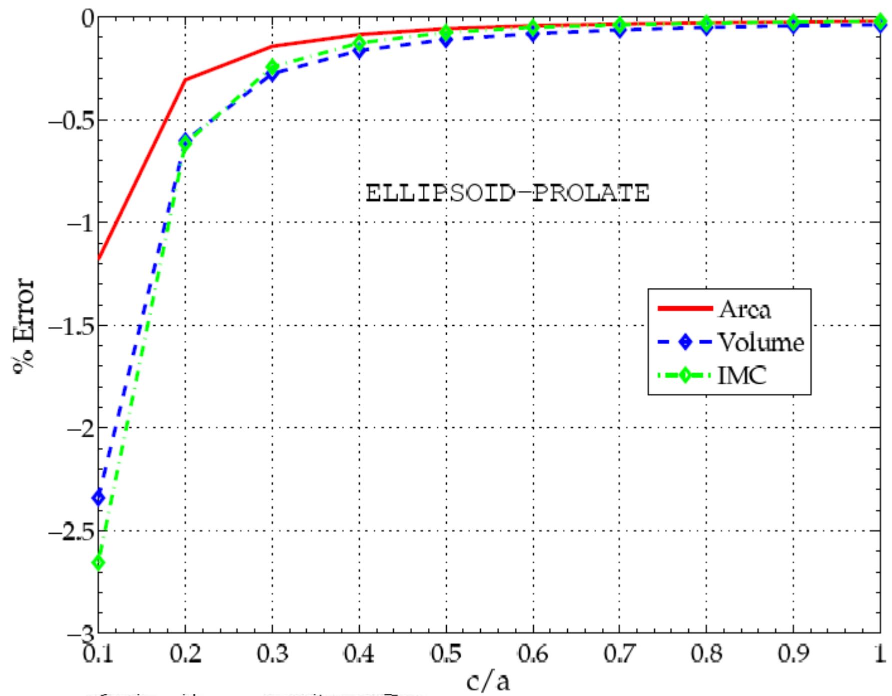
Tests with sphere

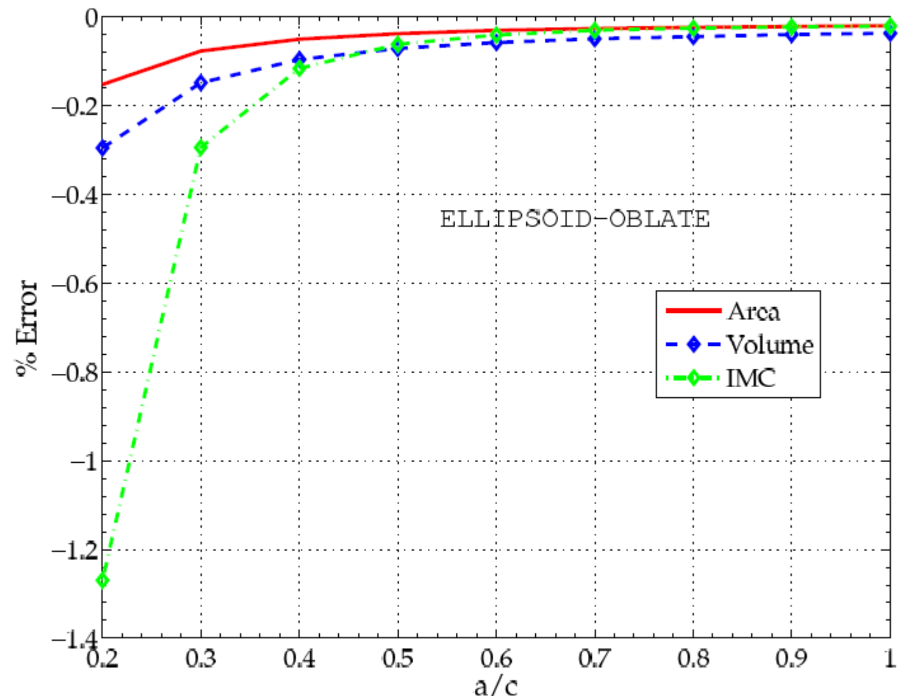
$$
\begin{aligned}
\rho(i,j,k) &= \begin{cases} \frac{\rho_0}{R} & (i,j,k) \neq (i_0,j_0,k_0) \\ \rho_0 & (i,j,k) = (i_0,j_0,k_0) \end{cases} \\
R &= \sqrt{(i-i_0)^2 + (j-j_0)^2 + (k-k_0)^2} \\
R &= \frac{\rho_0}{\rho_{th}}
\end{aligned}
$$

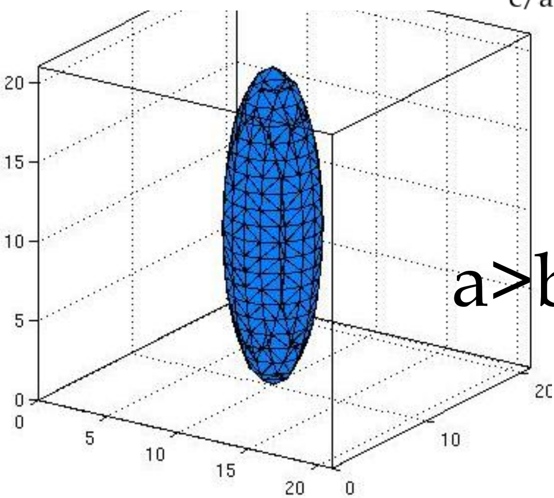




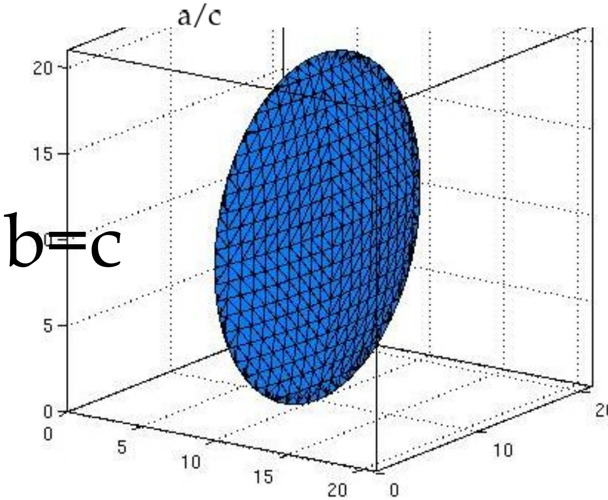
Test Ellipsoid





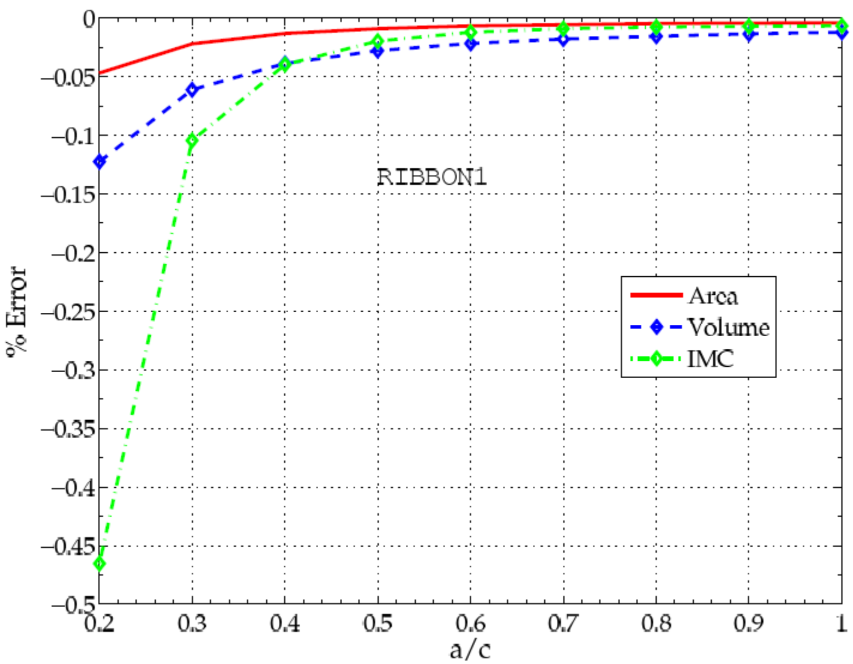


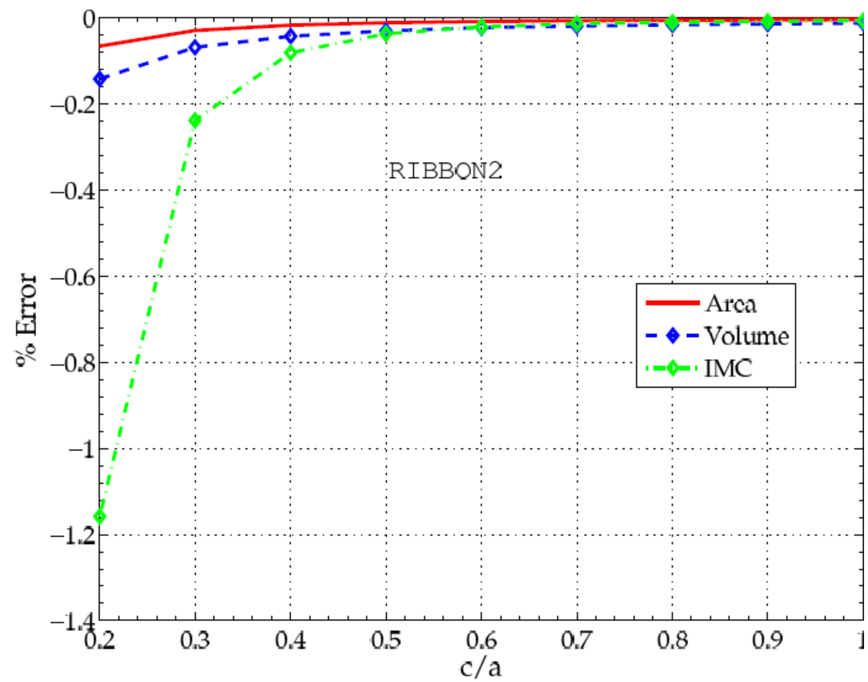
a>b=c



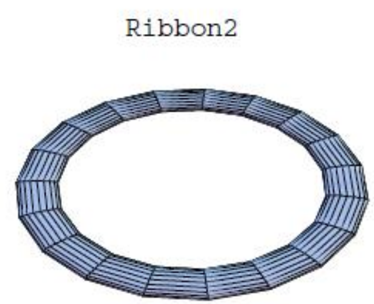
a<b=c

Test Continued …

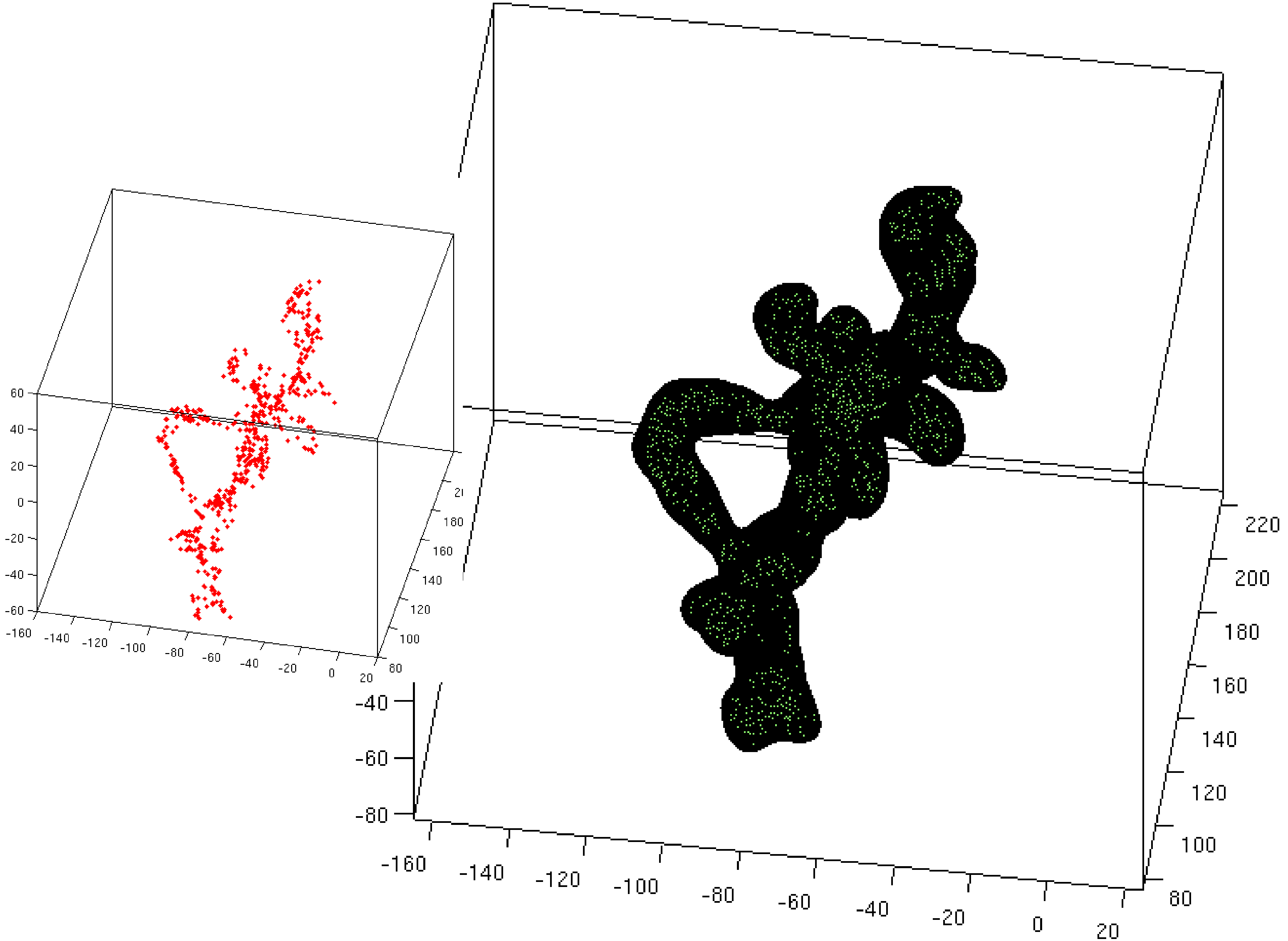




b>>a>c



b>>c>a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100
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140
160
180
200
220

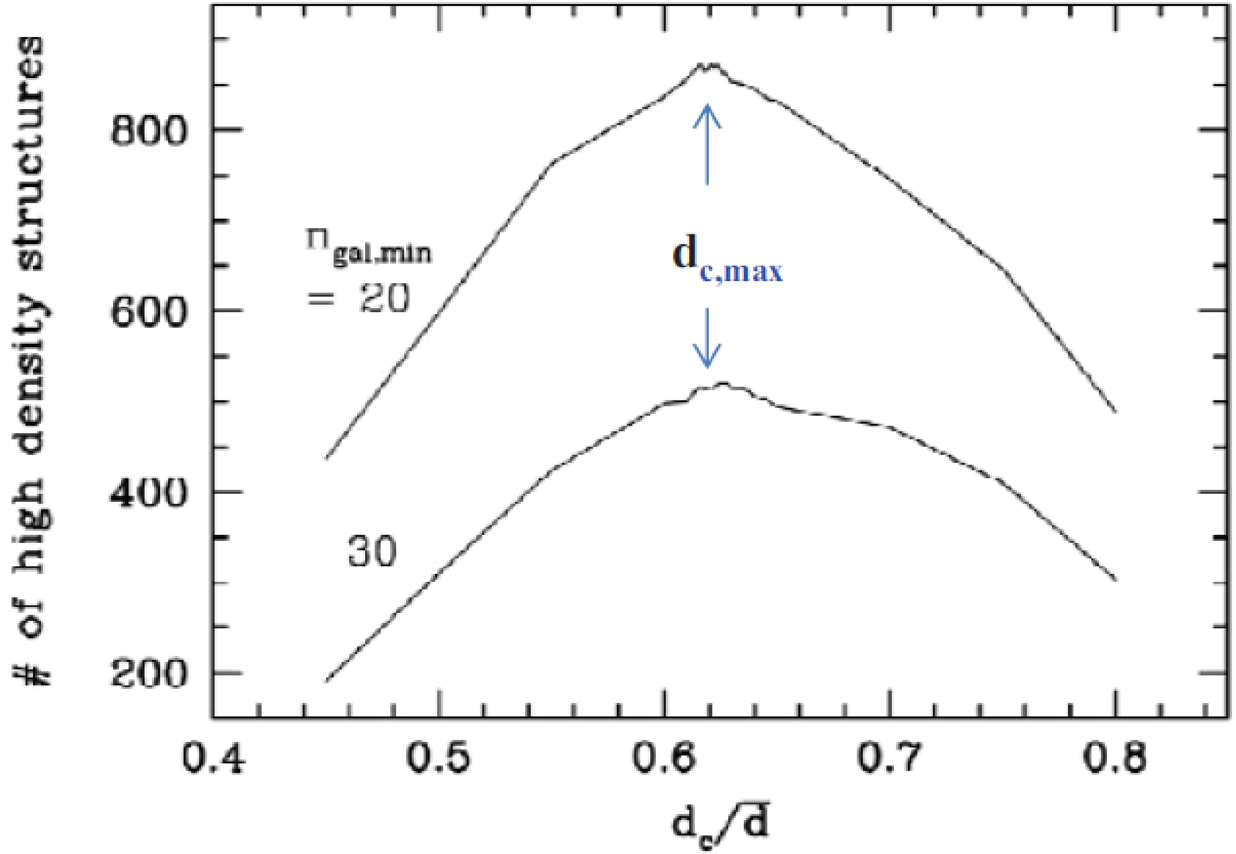
Applications

- Volume Limited subsample of SDSS Main galaxy sample.
  - N=116877 galaxies
  - M_r<-21.6
  - Mean separation $\bar{d}=9\,h^{-1}$ Mpc
- 200 mock catalog extracted from Horizon Run simulation having the same geometry and number density of the SDSS sample.

http://sdss.kias.re.kr/astro/Horizon-Run23/

- Identified structure using Friend-of-Friend algorithm of linking length d_c.

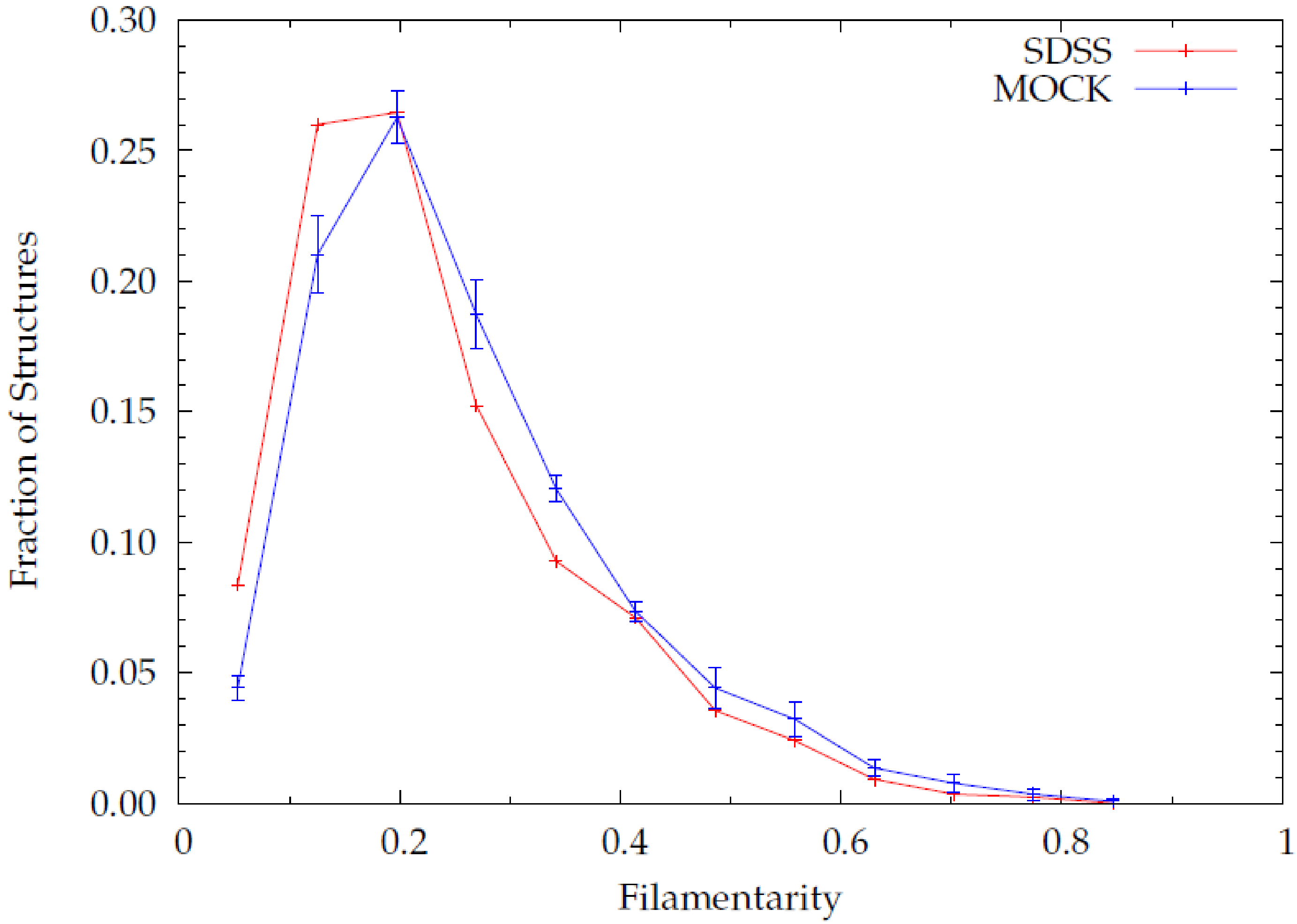
$d_c = 5.6\ h^{-1}$ Mpc



Each and every structures identified using FoF algorithm:

- is converted to density field using CIC algorithm of grid size 1 h^{-1}Mpc
- smoothed it with Gaussian Kernel of smoothing length $d_c/3 = 1.9\,h^{-1}$Mpc
- For applying shapefinder, we consider $\rho_{th} = (9/5.6)^3\bar{\rho} = 4.15\bar{\rho}$

Results



SDSS
MOCK
0.30
0.25
0.20
0.15
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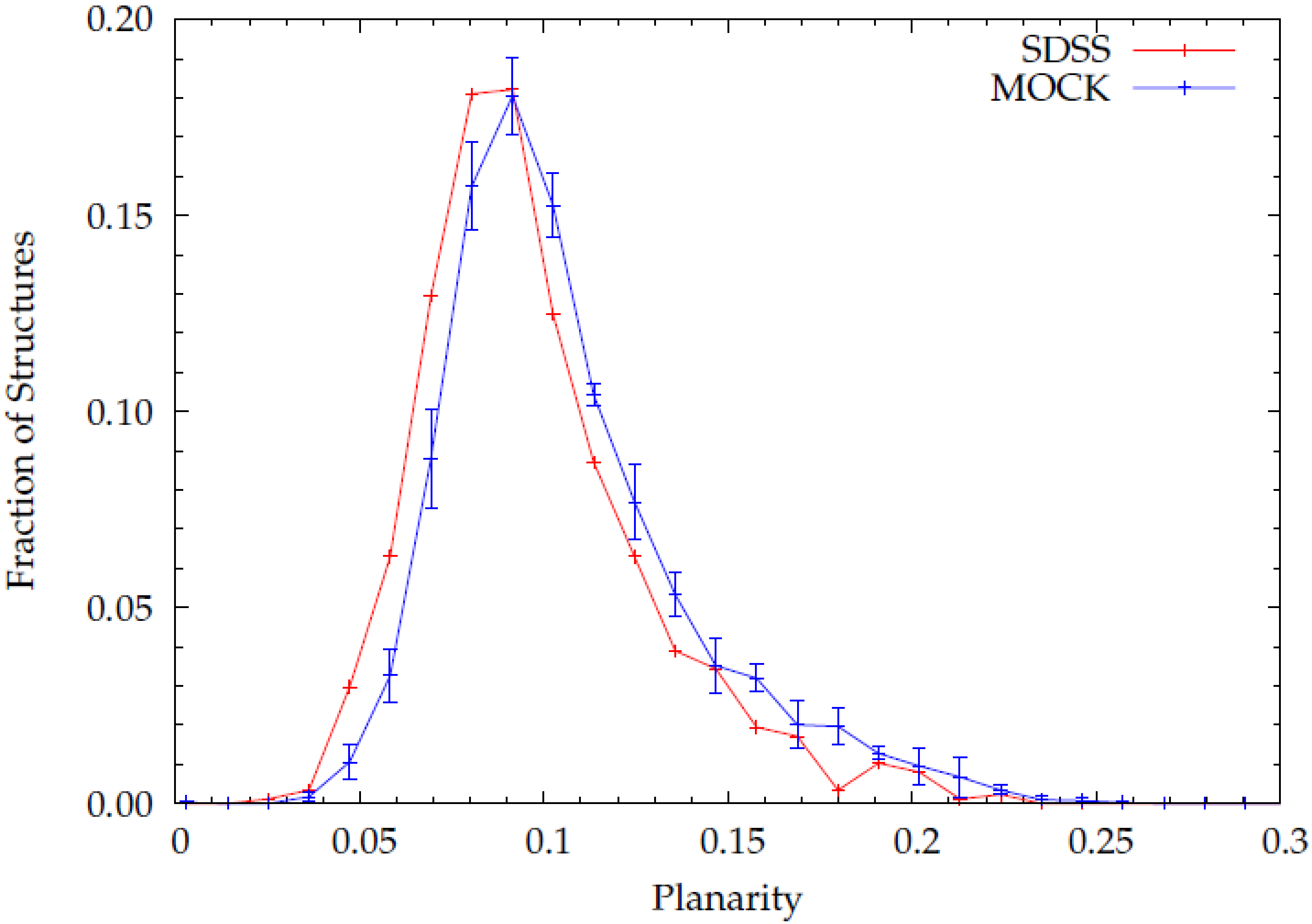
0.00
0
0.2
0.4
0.6
0.8
1
Fraction of Structures
Filamentarity

Results



SDSS
MOCK
0.20
0.15
0.10
0.05
0.00
Fraction of Structures
0
0.05
0.1
0.15
0.2
0.25
0.3
Planarity

Conclusions

- Minkowski Functionals and shapefinders are one of the robust method to identify individual structural elements in the Cosmic Web.
- SURFGEN produces the Minkowski Functionals and Shapefinders thus giving complete information about the shape and topology of the structures.
- Applying SURFGEN to SDSS DR7 we find the dominance of ribbon like structures.
- The difference in results of SDSS and Simulation is currently not known and require further analysis.

Thank you