ON THE INITIAL SHEAR FIELD OF THE COSMIC WEB

Graziano Rossi

Department of Astronomy and Space Science
Sejong University – Seoul, South Korea

The Zeldovich Universe – IAU Symposium 308

Tallinn, Estonia – June 24, 2014

OUTLINE

1. Dynamics of the cosmic web: key role of the initial shear field
2. '*Peak/dip picture*' of the cosmic web: new conditional distributions
3. Conditional shapes and '*peak/dip excursion-set based*' algorithm

4. Lyman-α forest, massive neutrinos, and the cosmic web

MAIN REFERENCES

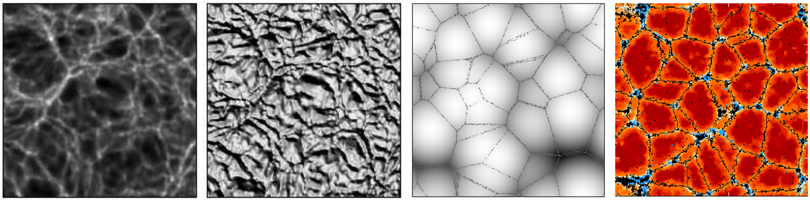
- **G. Rossi** et al. (2011), MNRAS, 416, 248
- **G. Rossi** (2012), MNRAS 421, 296-307
- **G. Rossi** (2013), MNRAS 430, 1486- 1503
- **G. Rossi** et al. (2014), A&A in press (arXiv:1401.6464)

Cosmic Web and Structure Formation

SF Theories

- Objects evolved from **peaks** in the initial field (Bardeen et al. 1986)
- Structure and clustering pattern of forming objects in the evolved nonlinear regime reflect initial conditions (Bond et al. 1991)

Key Literature $\rightarrow$ Zeldovich (1970), Doroshkevich (1970), Doroshkevich & Shandarin (1978), Kaiser (1984), Peebles (1984), Hoffman & Shaham (1984), Bardeen et al. (1986), Bond et al. (1986, 1991), Bertschinger (1987), Jones et al. (1988), van de Weygaert & Bertschinger (1996)



Aragon-Calvo et al. (2010)

Initial Shear Field

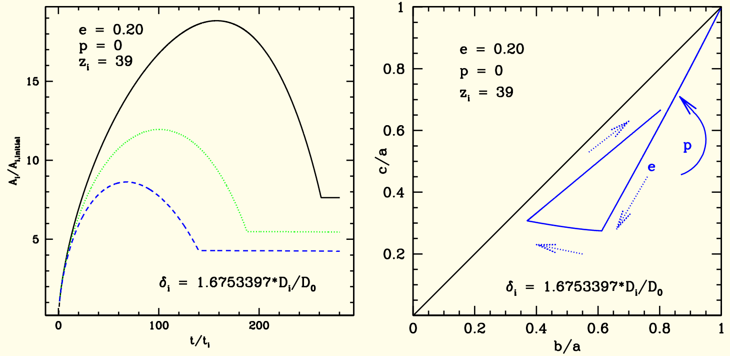
Web-like network shaped by **tidal field** $\rightarrow$ initial shear field plays a key role in shaping the cosmic web (as opposed to the inertia tensor)

- *Doroshkevich (1970)* $\rightarrow$ first to apply these methods to study the formation of cosmic structures
- Statistical properties of the eigenvalues of the shear tensor *(Doroshkevich & Shandarin 1978)* $\rightarrow$ pancakes
- Statistics of initial density field $\rightarrow$ classification of structures, mass function, merger trees, ...

Shear Tensor and Density Hessian

- **T** $\rightarrow$ shear tensor $\rightarrow$ related to gravitational field $\rightarrow$ eigenv. λ_i
- **H** $\rightarrow$ inertia tensor $\rightarrow$ related to density field $\rightarrow$ eigenv. ξ_i
- **T**|**H** $\rightarrow$ conditional shear given density Hessian$\rightarrow$ eigenv. ζ_i

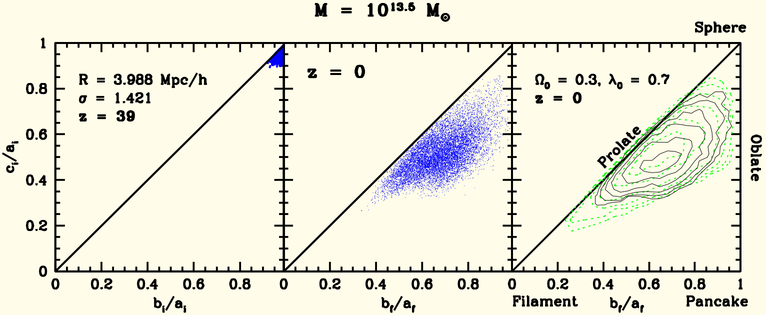
DYNAMICAL EVOLUTION



Rossi et al. (2011)

- **Ellipticity** $\rightarrow e_{\rm T} = \frac{\lambda_1 - \lambda_3}{2\delta_{\rm T}}$
- **Prolateness** $\rightarrow p_{\rm T} = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2\delta_{\rm T}}$

MODEL FOR HALO SHAPES



Rossi et al. (2011)

TWOFOLD PROCEDURE

- (1) **Nonlinear dynamics**: gravity → ellipsoidal collapse model
- (2) **Initial conditions**: statistics → excursion sets formalism

DOROSHKEVICH'S CELEBRATED FORMULA

UNCONDITIONAL DISTRIBUTION OF SHEAR EIGENVALUES

$$p(\lambda_1, \lambda_2, \lambda_3) = \frac{15^3}{8\sqrt{5}\pi} e^{-\frac{3}{2}(2k_1^2 - 5k_2)}(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)$$

$$\begin{aligned} k_1 &= \lambda_1 + \lambda_2 + \lambda_3 \equiv \delta_{\mathrm{T}} \\ k_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \end{aligned}$$

- Ordering of the eigenvalues is $\lambda_1 \geq \lambda_2 \geq \lambda_3$
- Derived by Doroshkevich (1970)
- Can derive partial distributions $\rightarrow p(\lambda_1), p(\lambda_2)$, etc.
- Neglects the fact that voids are maxima of the source displacement, or minima of the density field

JOINING TWO DESCRIPTIONS

Basic idea $\rightarrow$ Develop formalism which allows distinguishing between peaks/dips and random positions in space

- Need to consider the density Hessian, in addition to the shear field, and characterize their correlations
- Merge *excursion-set-based approach* + *peaks theory*

CONDITIONAL DISTRIBUTION OF TIDAL FIELD HAVING CURVATURE H

$$p(\mathbf{T}|\mathbf{H},\gamma) = \frac{15^3}{16\sqrt{5}\pi^3}\frac{1}{\sigma_{\mathrm{T}}^6(1-\gamma^2)^3}\mathrm{e}^{-\frac{3}{2\sigma_{\mathrm{T}}^2(1-\gamma^2)}(2K_1^2-5K_2)}$$

EXTENDED DOROSHKEVICH'S FORMALISM

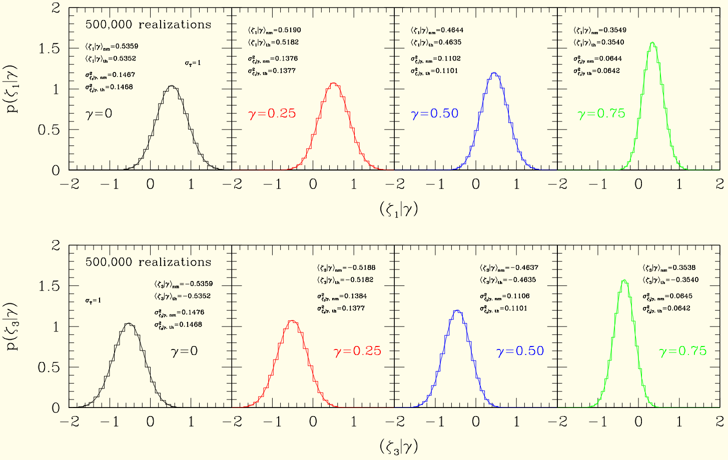
CONDITIONAL DISTRIBUTION OF SHEAR EIGENVALUES

$$p(\zeta_1, \zeta_2, \zeta_3|\gamma) \equiv p(\lambda_1, \lambda_2, \lambda_3|\xi_1, \xi_2, \xi_3, \gamma)$$
$$= \frac{15^3}{8\sqrt{5}\pi} \frac{1}{(1-\gamma^2)^3} \mathrm{e}^{-\frac{3}{2(1-\gamma^2)}(2K_1^2 - 5K_2)} (\zeta_1 - \zeta_2)(\zeta_1 - \zeta_3)(\zeta_2 - \zeta_3)$$

G. Rossi (2012) – MNRAS 421, 296-307

$$\begin{aligned} K_1 &= \zeta_1 + \zeta_2 + \zeta_3 = k_1 - \gamma h_1 \\ K_2 &= \zeta_1\zeta_2 + \zeta_1\zeta_3 + \zeta_2\zeta_3 = k_2 + \gamma^2 h_2 - \gamma h_1 k_1 + \gamma\tau \\ \tau &= \lambda_1\xi_1 + \lambda_2\xi_2 + \lambda_3\xi_3 \\ k_1 &= \lambda_1 + \lambda_2 + \lambda_3, \quad k_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \\ h_1 &= \xi_1 + \xi_2 + \xi_3, \quad h_2 = \xi_1\xi_2 + \xi_1\xi_3 + \xi_2\xi_3 \\ \zeta_{\rm i} &\equiv (\lambda_{\rm i}|\gamma, \xi_{\rm i}) = \lambda_{\rm i} - \gamma\xi_{\rm i} \end{aligned}$$

Conditional Distributions



G. Rossi (2012) – MNRAS 421, 296-307

Shape Parameters: Extensions

New formalism (*density web* vs *strain web*) which combines

- Bardeen et al. (1986) $\rightarrow$ density $\rightarrow$ density Hessian
- Bond & Myers (1996) $\rightarrow$ gravity $\rightarrow$ shear

G. Rossi (2013) – MNRAS 430, 1486- 1503

Density $\rightarrow$ Hessian

$$e_{\rm H} = \frac{\xi_1 - \xi_3}{2\delta_{\rm H}}, \; p_{\rm H} = \frac{\xi_1 + \xi_3 - 2\xi_2}{2\delta_{\rm H}}$$

Gravity $\rightarrow$ Shear

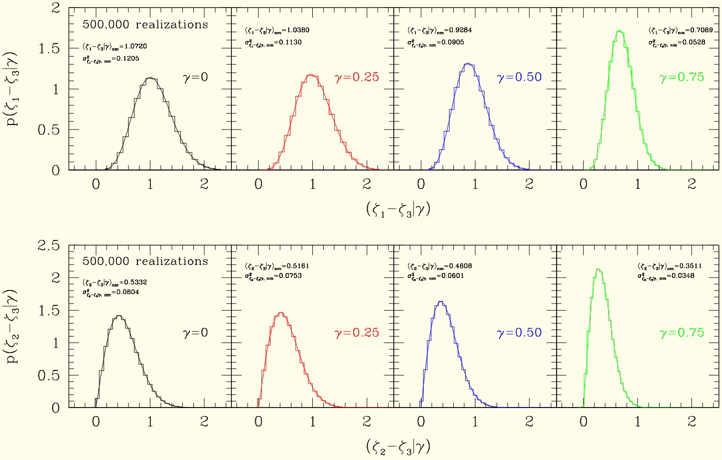
$$e_{\rm T} = \frac{\lambda_1 - \lambda_3}{2\delta_{\rm T}}, \; p_{\rm T} = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2\delta_{\rm T}}$$

Extended Formalism

$$\Delta_{\rm T|H} E_{\rm T|H} = \frac{\zeta_1 - \zeta_3}{2} = \delta_{\rm T} e_{\rm T} - \gamma \delta_{\rm H} e_{\rm H}, \qquad \Delta_{\rm T|H} P_{\rm T|H} = \frac{\zeta_1 + \zeta_3 - 2\zeta_2}{2} = E_{\rm T|H} - (\zeta_2 - \zeta_3)$$

- Can express all in terms of unconditional relations
- Can express the conditional joint probability of initial halo/void shapes

CONDITIONAL ELLIPTICITY & PROLATENESS



G. Rossi (2013) – MNRAS 430, 1486-1503

THE 'PEAK-DIP EXCURSION-SET-BASED' ALGORITHM

THE EXCURSION SET APPROACH

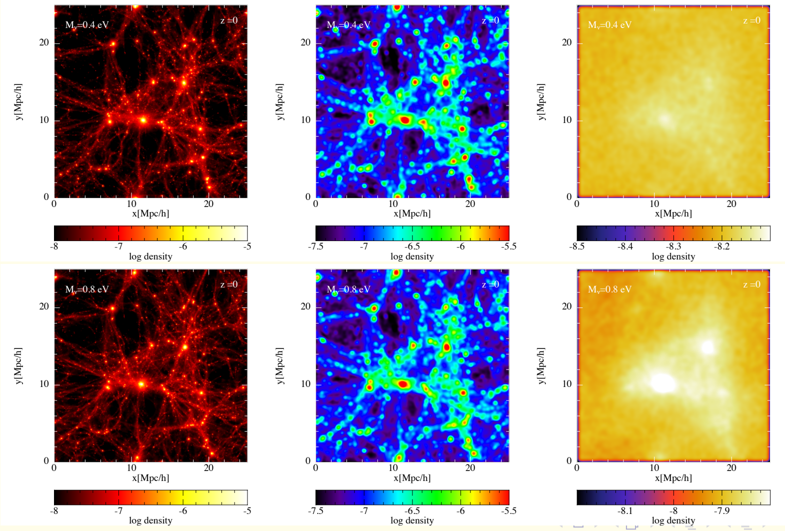
Pick a particle at random and smooth the linear density field over ever smaller spheres around it, until the criterion for collapse at some redshift is satisfied. The mass in the sphere is then identified as that of the collapsed object to which the particle belongs

- Statistics of the four-dimensional field $F(\mathbf{r}, R_{\rm f})$
- Trajectories of the field as a function of the filter radius at a fixed position (with F the linear overdensity)
- Rate at which random trajectories meet an absorbing barrier $\rightarrow$ mass function
- Bound structures forming at t are regions above some initial critical overdensity $F_{\rm cr}$
- A diffusion-like problem
- More rigorous treatment $\rightarrow$ see Maggiore & Riotto (2010)

EXTENSION TO PEAKS/DIPS $\rightarrow$ ROSSI (2012, 2013)

- Determine $\delta_{\rm T}, e_{\rm T}, p_{\rm T}$ which are simple combinations of the eigenvalues
- Check if $(\delta_{\rm T}, e_{\rm T}, p_{\rm T})$ cross the barrier $B(\delta_{\rm T}, e_{\rm T}, p_{\rm T})$ at the mass-scale $\sigma_{\rm T}$
- If so exit – if not, continue the loop

LYMAN-α FOREST AND MASSIVE NEUTRINOS



PRODUCTS

A suite of 48 hydrodynamical simulations with massive neutrinos

- Typical set (3 sims.) $\rightarrow$ 100 h^{-1}Mpc$/768^3$ (a), 25 h^{-1}Mpc$/768^3$ (b), 25 h^{-1}Mpc$/192^3$ (c)
- With splicing technique $\rightarrow$ equivalent of 100 h^{-1}Mpc$/3072^3$
- Full snapshots at a given redshift ($z = 4.6 - 2.2$, $\Delta z = 0.2$)
- 100,000 quasar sightlines per redshift interval per simulation

Group I

Simulation Set	M_ν [eV]
BG a/b/c	0
NUBG a/b/c	0.01
NU01 a/b/c	0.1
NU01-norm a/b/c	0.1
NU02 a/b/c	0.2
NU03 a/b/c	0.3
NU04 a/b/c	0.4
NU04-norm a/b/c	0.4
NU08 a/b/c	0.8
NU08-norm a/b/c	0.8

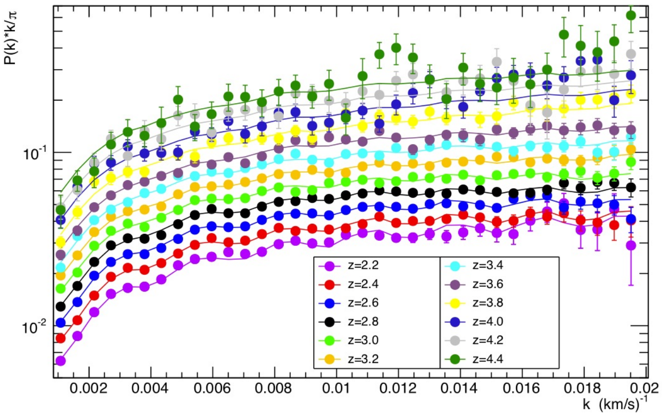
Group II

Simulation Set	M_ν [eV]
γ+NU08 a/b/c	0.8
H_0+NU08 a/b/c	0.8
n_s+NU08 a/b/c	0.8
Ω_m+NU08 a/b/c	0.8
σ_8+NU08 a/b/c	0.8
T_0+NU08 a/b/c	0.8

G. Rossi et al. (2014), A&A (arXiv:1401.6464)

- **Group I** $\rightarrow$ Best-guess and neutrino runs
- **Group II** $\rightarrow$ Cross-terms

1D LYA BOSS POWER SPECTRUM



Three Main Messages

1. New theoretical framework and formulae for characterizing the initial shear field

2. New '*peak-dip excursion-set-based*' algorithm

3. Novel suite of hydrodynamical simulations for the Lyman-α forest with massive neutrinos

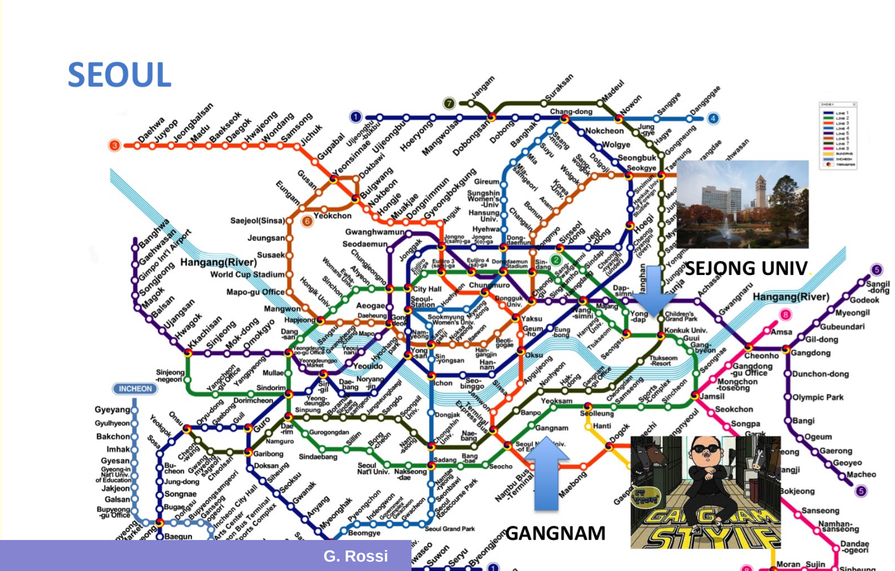
Eq. (47) – G. Rossi (2012), MNRAS 421, 296-307

$$p(\zeta_1, \zeta_2, \zeta_3 | \gamma) = \frac{15^3}{8\sqrt{5}\pi} \frac{1}{(1-\gamma^2)^3} \exp\left[-\frac{3(2K_1^2 - 5K_2)}{2(1-\gamma^2)} \right] (\zeta_1 - \zeta_2)(\zeta_1 - \zeta_3)(\zeta_2 - \zeta_3)$$

VISIT SEJONG UNIVERSITY IN SEOUL!



SEJONG UNIVERSITY IS CLOSE TO GANGNAM!



Basic Notation (1)

- Ψ displacement field
- Φ potential of the displacement field
- S_Ψ source of the displacement field
- $\mathbf{q}$ Lagrangian coordinate, $\mathbf{x}$ Eulerian coordinate,
- T_{ij} shear tensor of disp. field $\rightarrow$ eigen. $\lambda_1 \geq \lambda_2 \geq \lambda_3$, rms σ_{T}
- H_{ij} Hessian matrix $\rightarrow$ eigen. $\xi_1 \geq \xi_2 \geq \xi_3$, rms σ_{H}
- J_{ij} Jacobian of the displacement field $(i, j = 1, 2, 3)$

$$\mathbf{x}(\mathbf{q}) = \mathbf{q} + \Psi(\mathbf{q})$$

$$J_{ij}(\mathbf{q}) = \frac{\partial x_i}{\partial q_j} = \delta_{ij} + T_{ij}$$

$$T_{ij} = \frac{\partial \Psi_i}{\partial q_j} = \frac{\partial^2 \Phi}{\partial q_i \partial q_j}, \quad H_{ij} = \frac{\partial^2 S_\Psi}{\partial q_i \partial q_j}$$

$$S_\Psi(\mathbf{q}) = \sum_{i=1}^{3} \frac{\partial \Psi_i}{\partial q_i} \equiv \sum_{i=1}^{3} \frac{\partial^2 \Phi}{\partial q_i^2}$$

Basic Notation (2)

Φ Gaussian random field determined by $P(k)$, k wave number, $W^2(k)$ additional smoothing window function. Density field described by S_Ψ, also Gaussian. Correlations between these fields:

$$\langle T_{ij} T_{kl} \rangle = \frac{\sigma_T^2}{15} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\langle H_{ij} H_{kl} \rangle = \frac{\sigma_H^2}{15} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\langle T_{ij} H_{kl} \rangle = \frac{\Gamma_{TH}}{15} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\sigma_T^2 = S_2 \equiv \sigma_0^2,\ \sigma_H^2 = S_6 \equiv \sigma_2^2,\ \Gamma_{TH} = -S_4 \equiv -\sigma_1^2$$

$$S_n = \frac{1}{2\pi^2} \int_0^\infty k^n\, P(k)\, W^2(k) \mathrm{d}k$$

$$\sigma_j^2 = \frac{1}{2\pi^2} \int_0^\infty k^{2(j+1)}\, P(k)\, W^2(k)\, \mathrm{d}k \equiv S_{2(j+1)}$$

$$\gamma = \Gamma_{TH}/\sigma_T \sigma_H = \frac{\sigma_1^2}{\sigma_0 \sigma_2} \equiv = \gamma_{\mathrm{BBKS}}$$