IAU-308 Zel'dovich Universe — Tallinn — 24 June 2014

Disentangling Cosmic Web using Lagrangian Submanifold

S. Shandarin (Kansas)
M. Medvedev (Kansas)
J. Hidding (Groningen)

Motivation

- **Formation of the Large-Scale Structure**
  - identification of:
    - *voids*
    - *walls (pancakes)*
    - *filaments*
    - *halos*
  - relevant to:
    - *Ly-alpha forest*
    - *galaxy formation in voids*

- **Dark matter distribution on small scales**
  - fine-grained distribution function of DM
  - identification / counting of:
    - *caustics*
    - *streams (e.g., tidal streams)*
  - relevant to:
    - *direct and indirect detection experiments (e.g., "boost factor" for DM annihilation)*
    - *cosmic archeology (e.g., dwarfs & streams in the Local Group)*

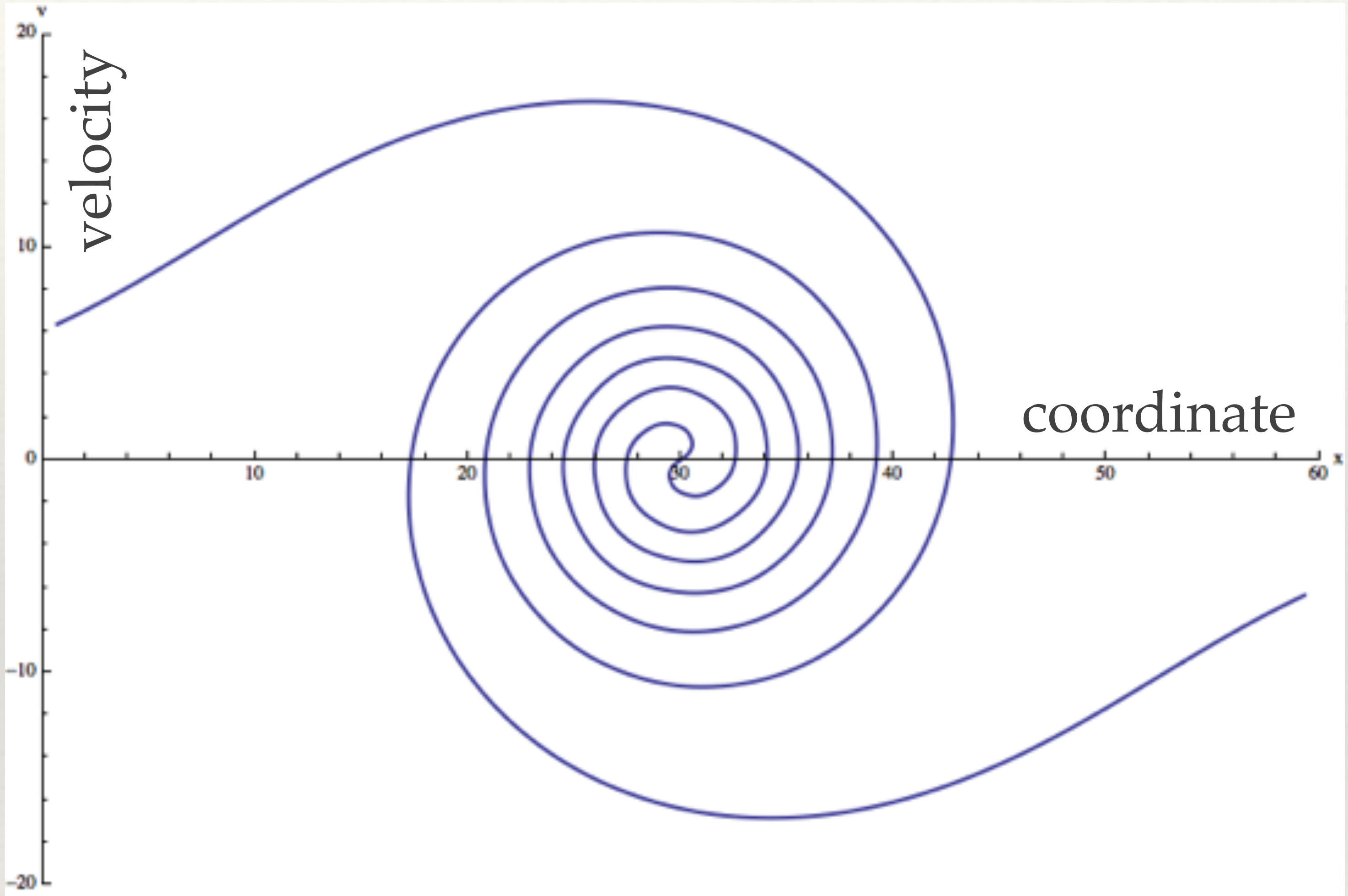
Collapse of an overdensity (1D example)

Phase space:

- contains all information about system's dynamics,

  but

- all projections onto 3D are *multivalued* and contain caustics
- the space is *non-metric*
- numerically, **v**, being a derivative, is more noisy than **x**

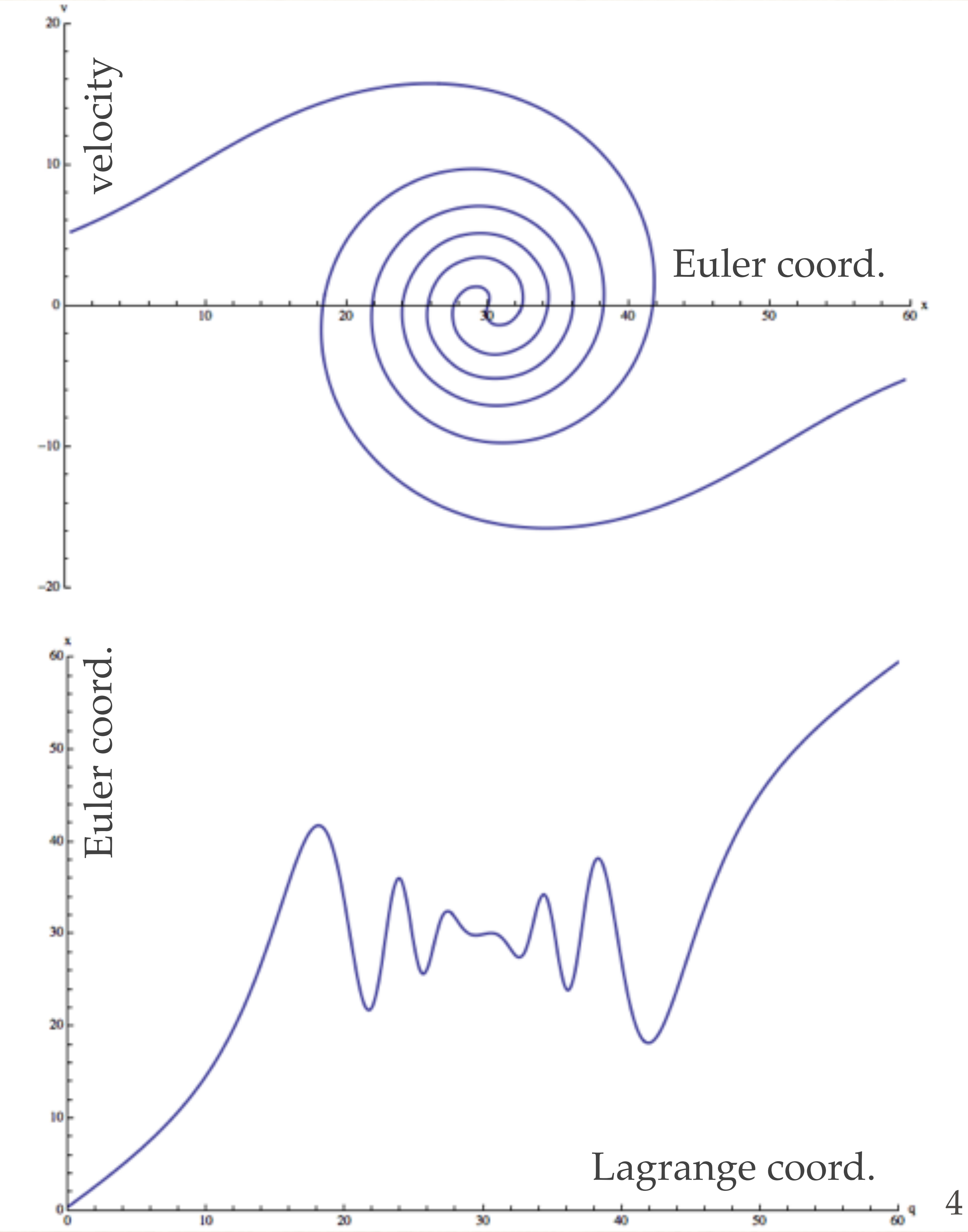


Phase space vs. Lagrange submanifold

Equivalently, one can use the **Lagrangian submanifold**:

- $\mathbf{x} = \mathbf{x}(\mathbf{q})$
  $\mathbf{x}$ — Eulerian coord
  $\mathbf{q}$ — Lagrangian coord

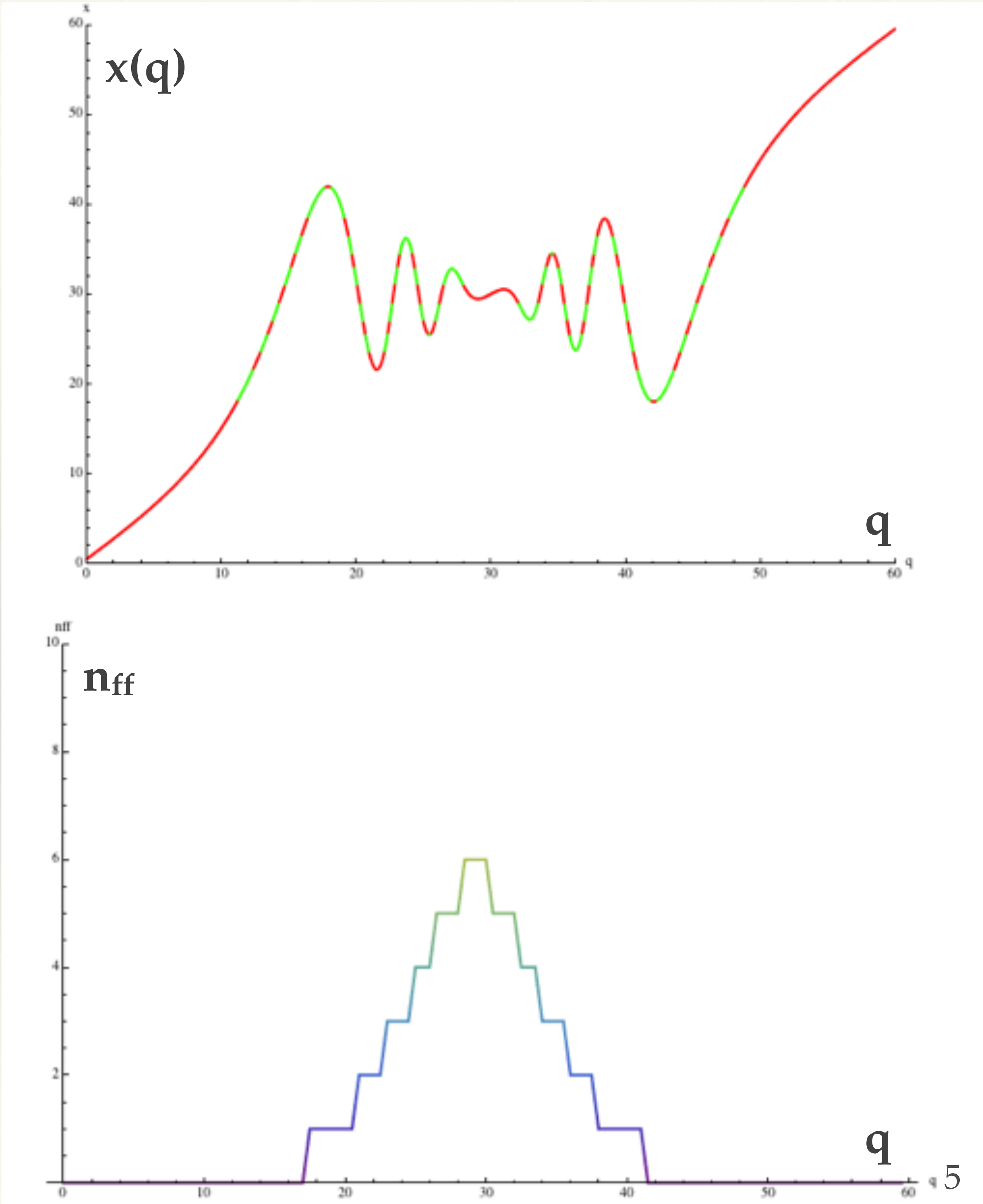
- Dynamically equivalent to phase space



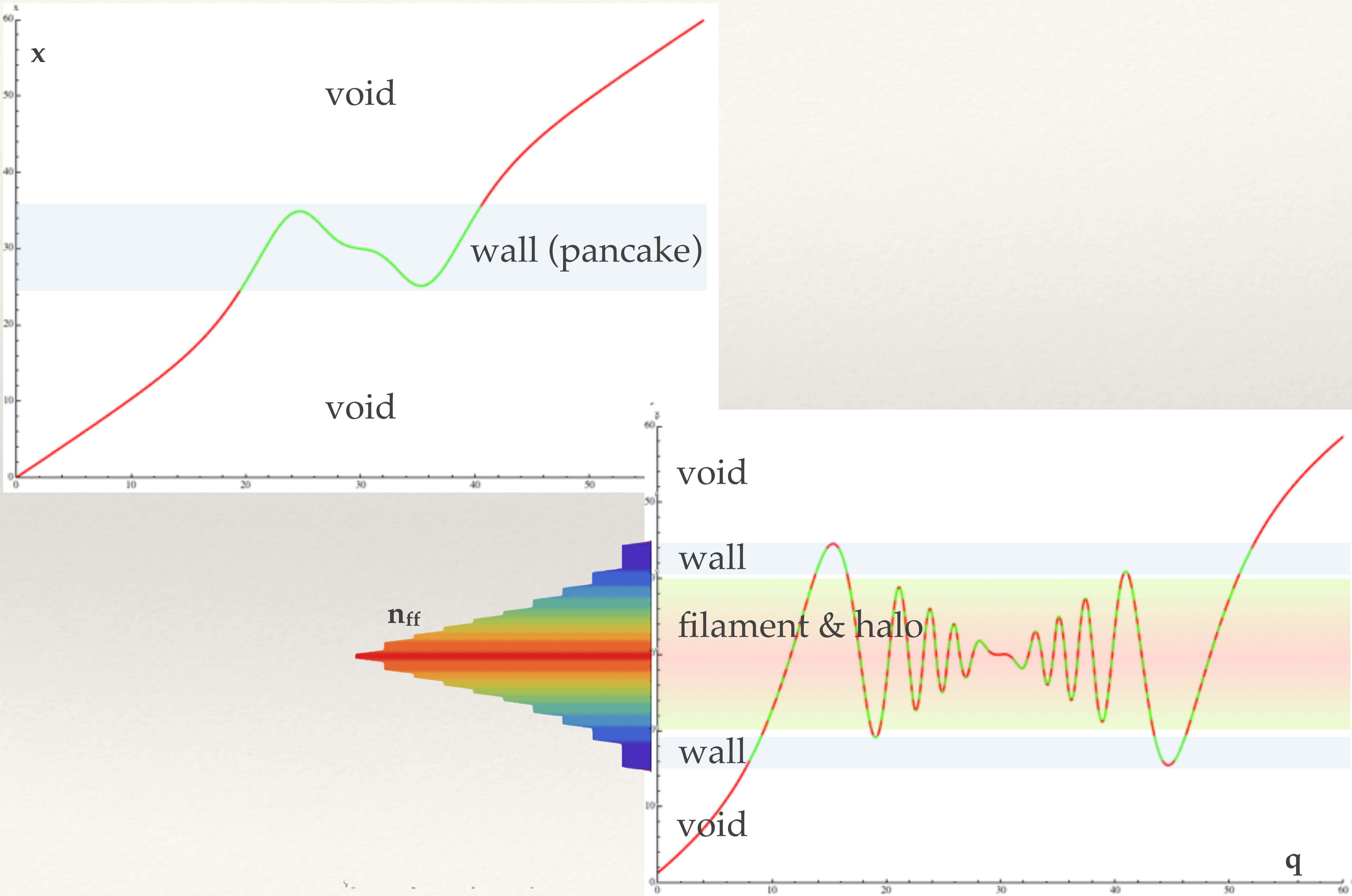
Lagrange submanifold

Advantages of
Lagrangian submanifold (LS):

- single-valued mapping (*epimorphism*)
- *metric* space
- numerically, it is less noisy than phase space (**q** is known exactly)
- count "flip-flops" (or "flow U-turns")
- much easier to analyze & to find structures: *voids, walls, filaments, halos, substructure, streams*

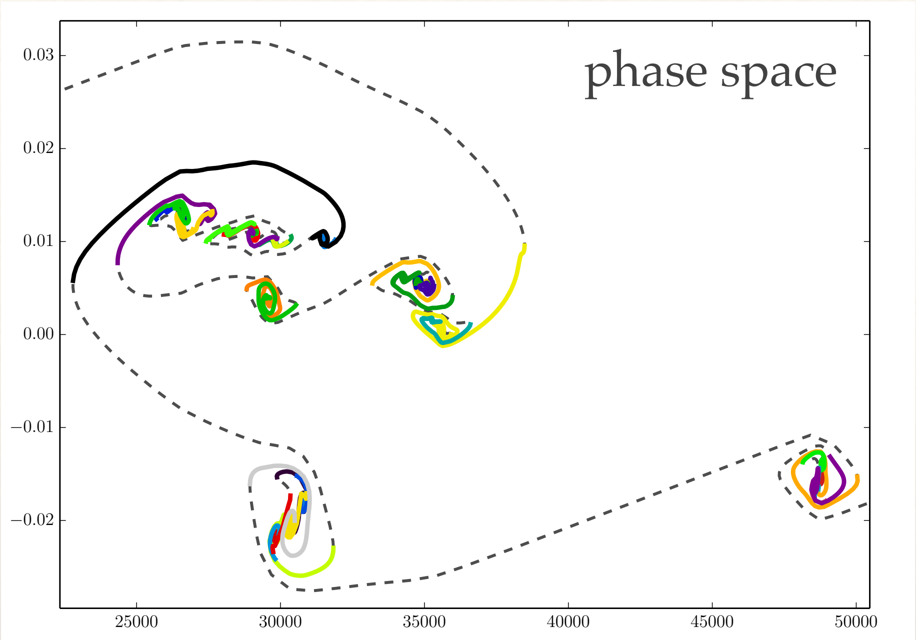


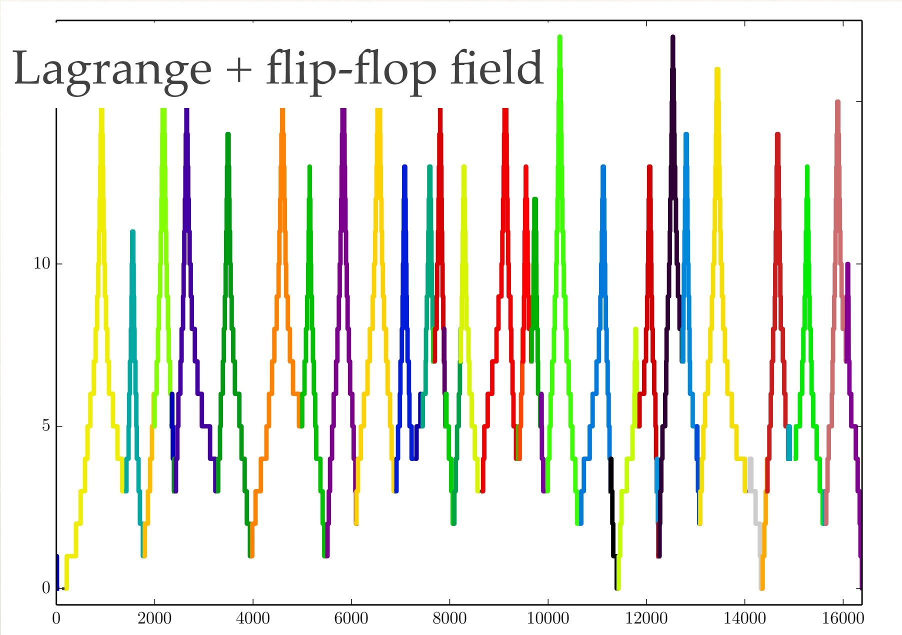
Disentangling the structure with LS



Collapse of a gaussian field (1D & 2D)

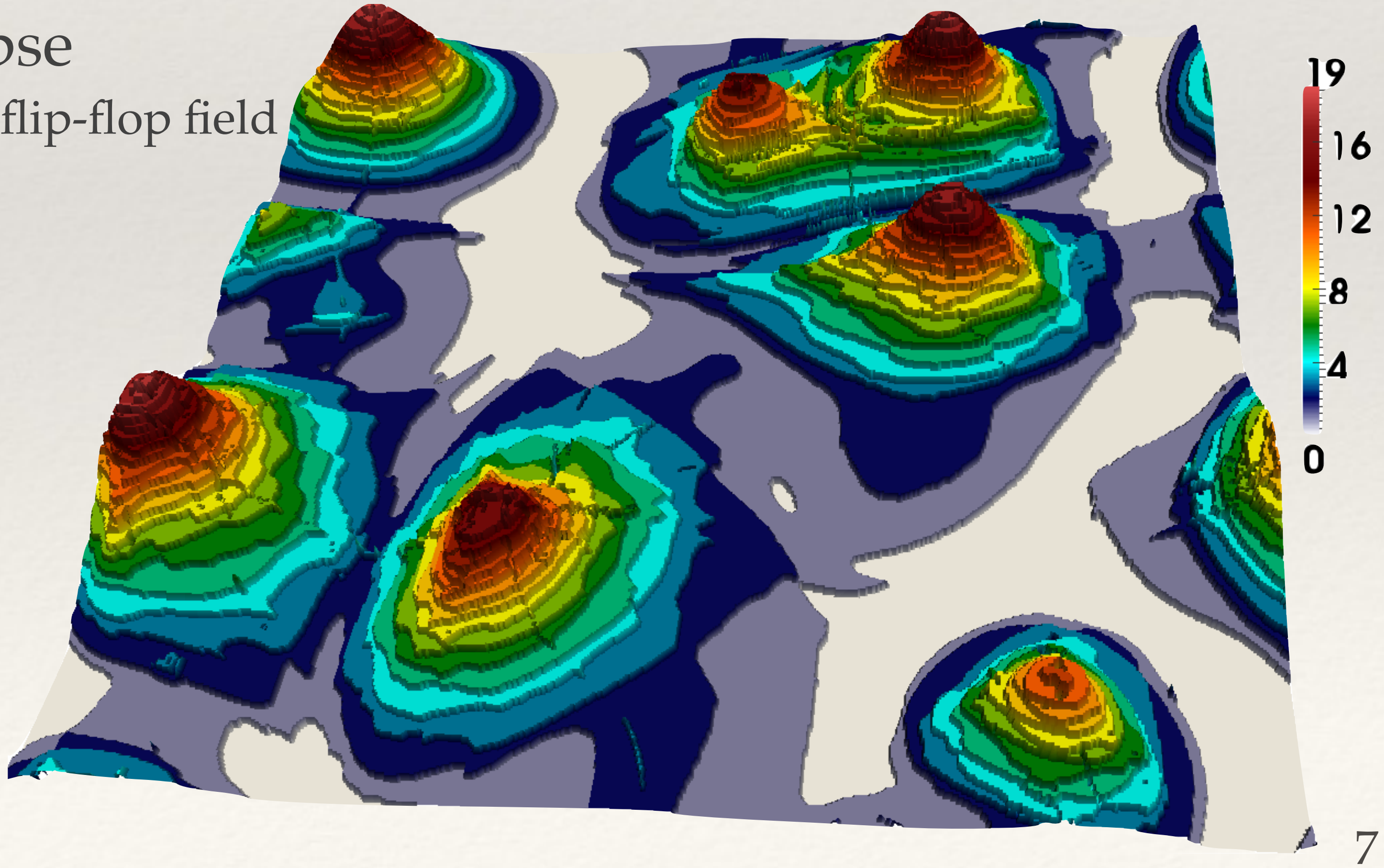
1D collapse





2D collapse

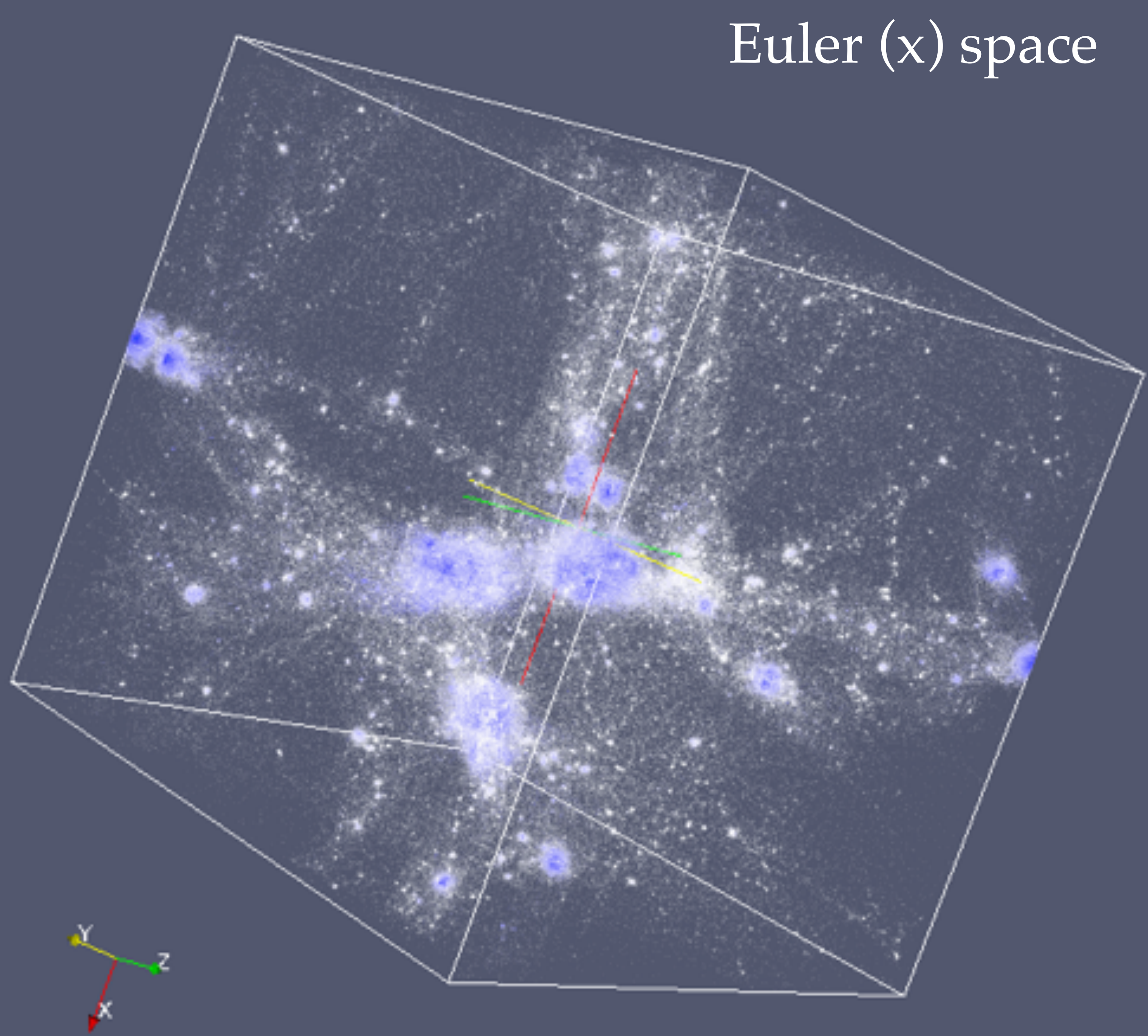
Lagrange + flip-flop field

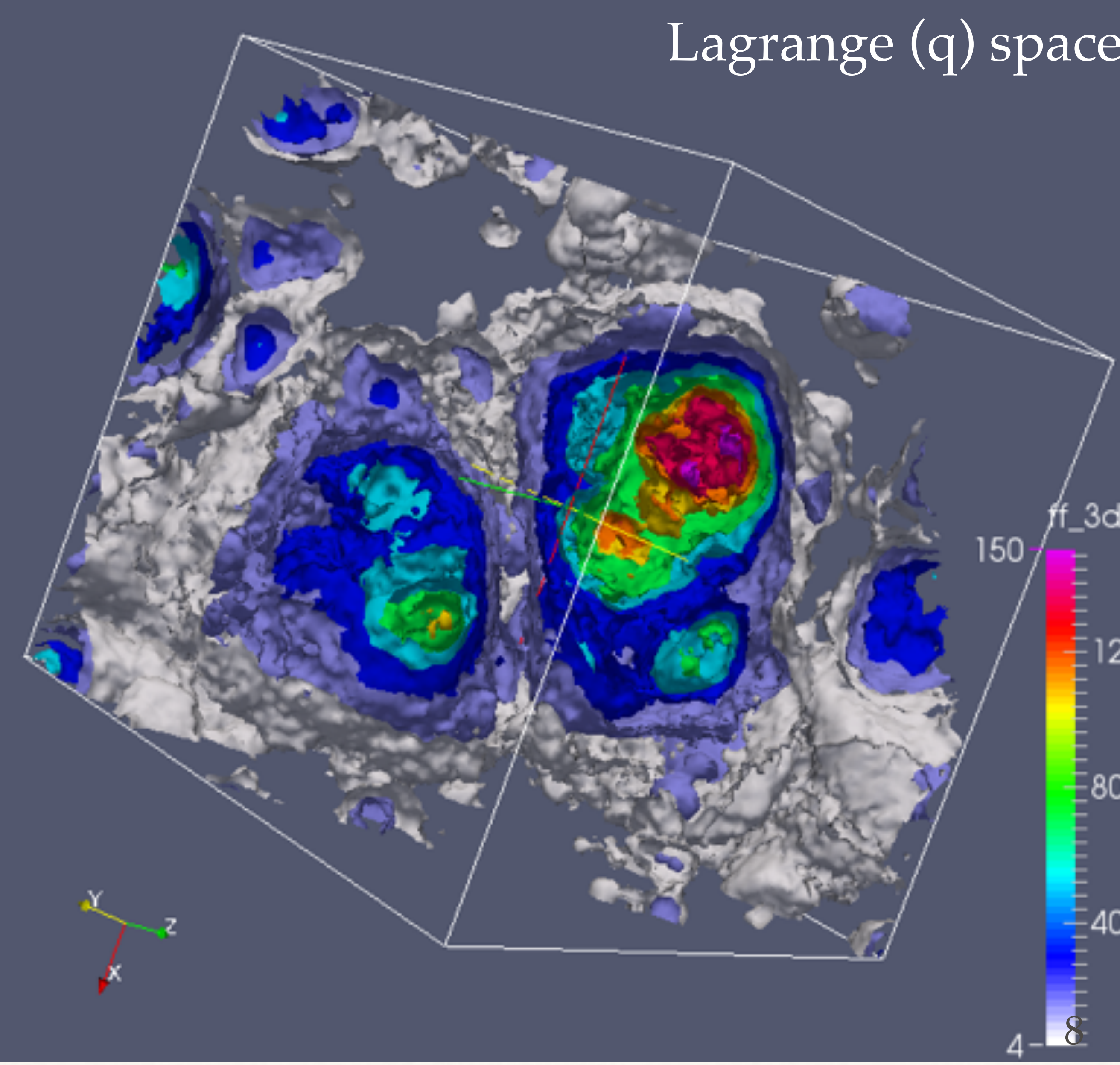


Unlike other stream-counting algorithms, the use of Lagrange submanifold allows one to disentangle substructure individually — no projection effects!

Structure formation in ΛCDM

Zoomed-in Gadget simulations: 1 Mpc/h, 256^3
flip-flops computed for each particle at each time-step

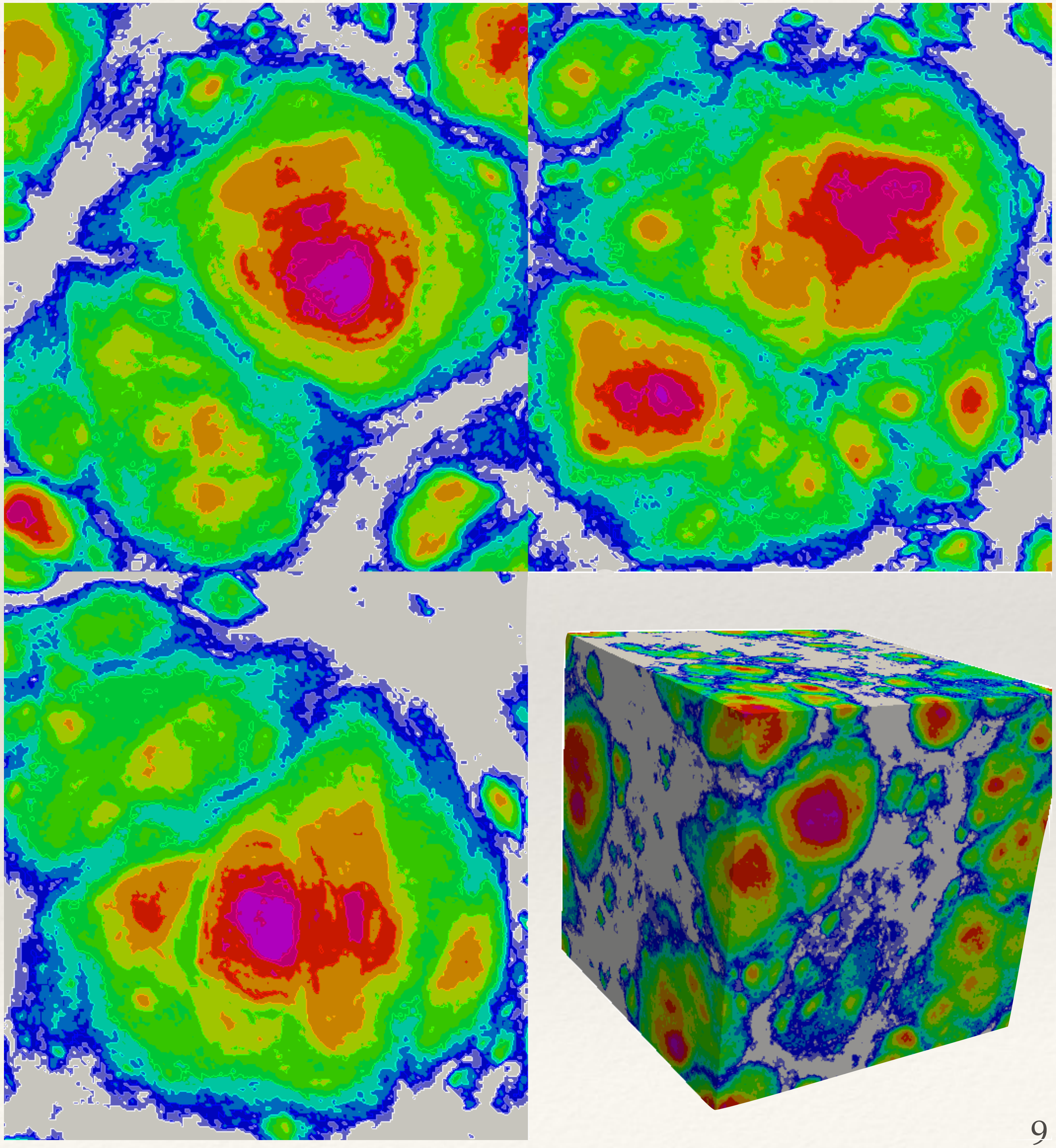




Structure formation in ΛCDM (q-space)

The 1 Mpc/h simulation cube in Lagrangian space color-coded by #of flip-flops

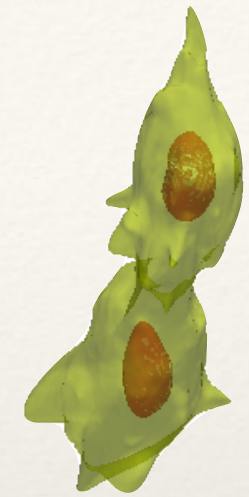
Topology of the structure: constant $\mathbf{n}_{ff}$ contours never cross each other — substructure is imbedded in a larger structure — as in *matryoshka doll*



Identifying the structure

Halos

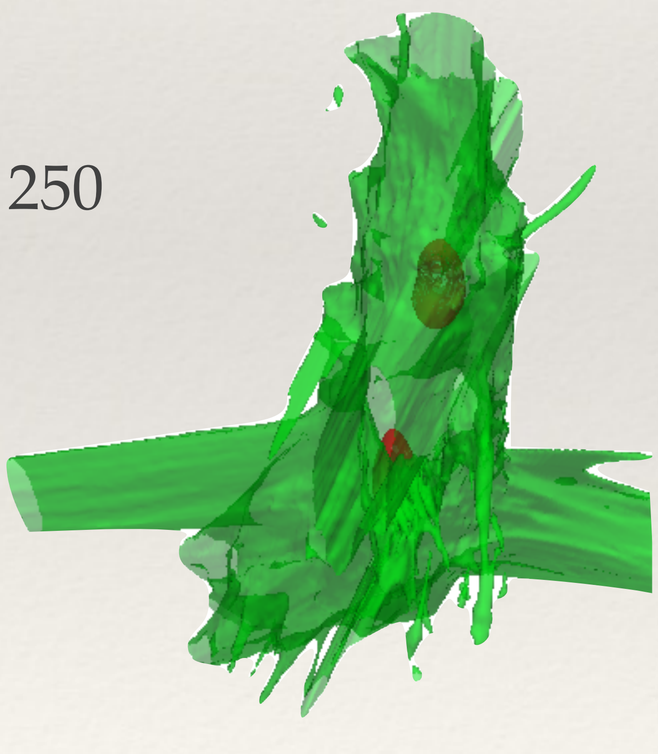
$n_{streams}$ = 50,000 & 15,000

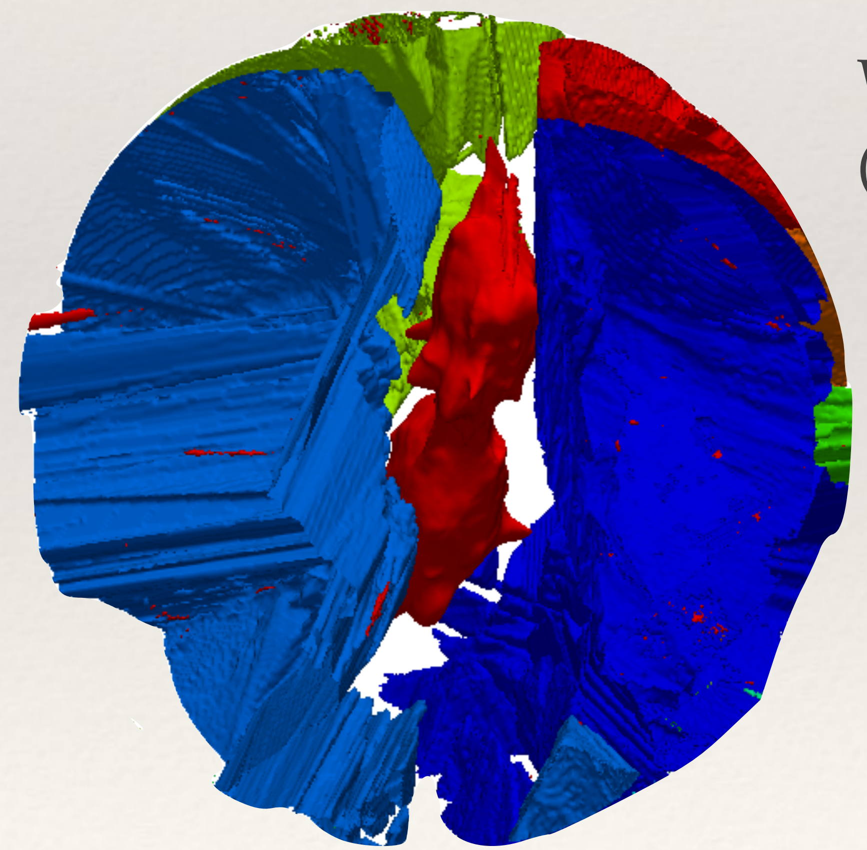


$n_{streams}$ = 15,000 & 5,000

Filaments

$n_{streams}$ = 250





Walls (pancakes)

$n_{streams}$ = 3

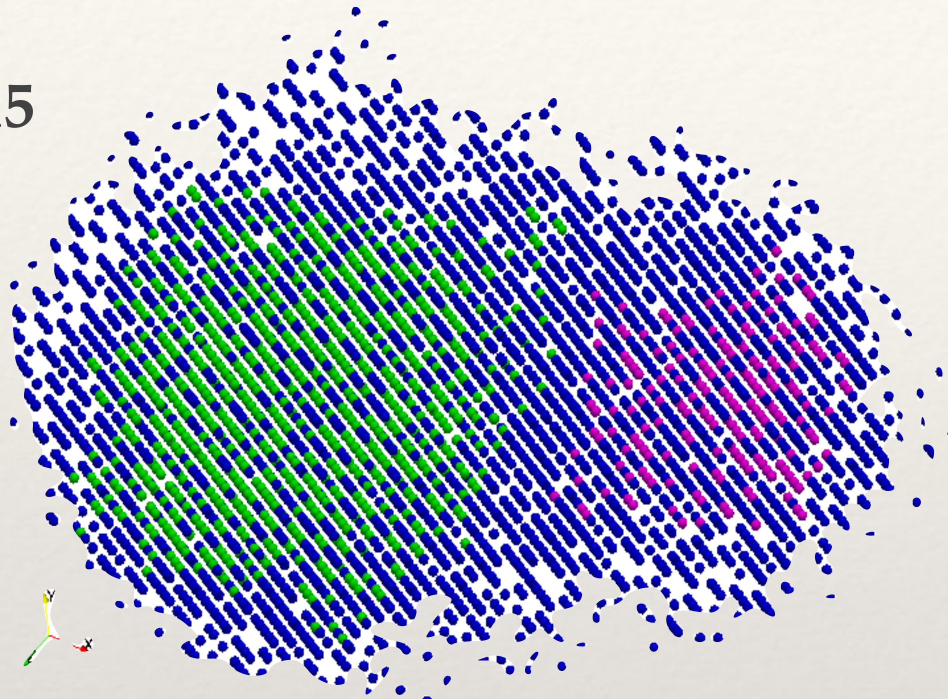
Euler space plot; R=300 kpc

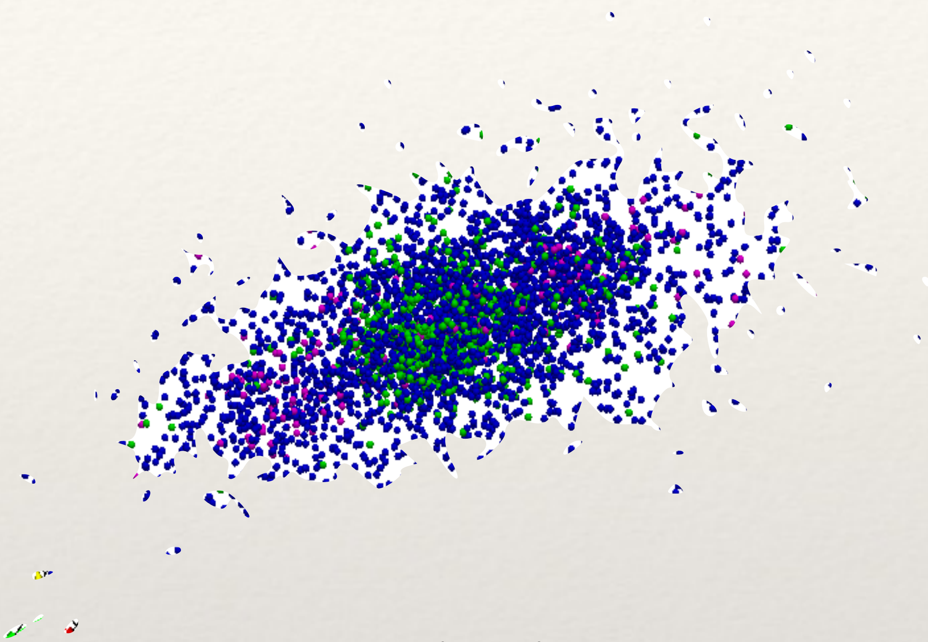
Disentangling the substructure

Lagrange space

Euler space

$\mathbf{n_{ff}} = \mathbf{15}$





projection: $\mathbf{q} \longmapsto \mathbf{x}$

$\mathbf{n_{streams}} = 1 + 2\ \mathbf{\Sigma}\ \mathbf{n_{ff,substructure}}$

$\mathbf{n_{ff}} = \mathbf{25}$

Disentangling by velocities?

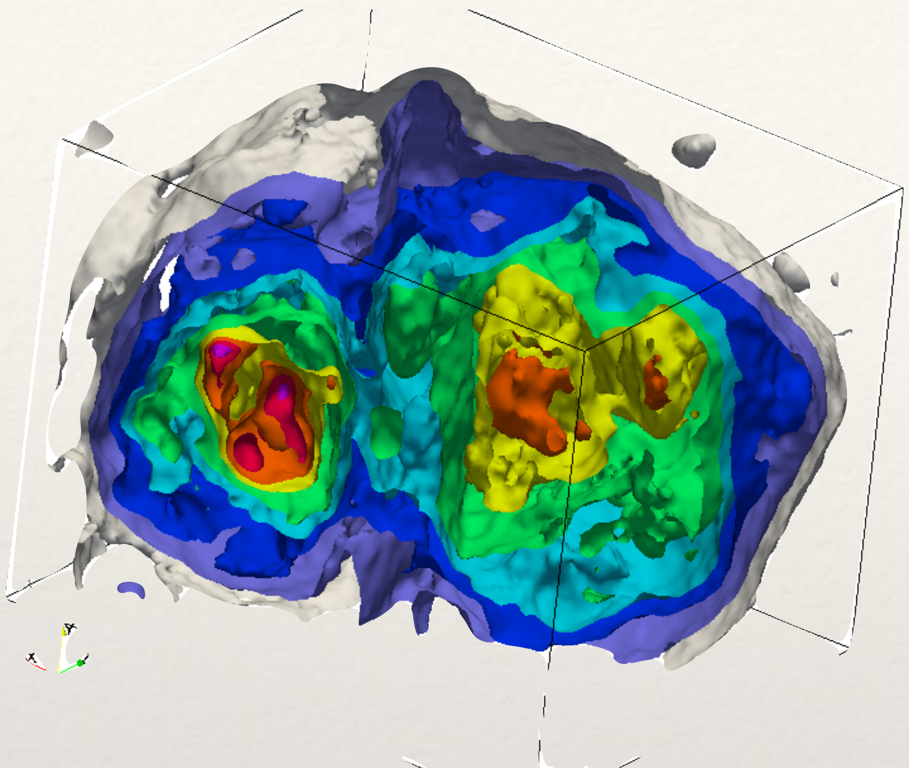
Velocity space (2 projections)

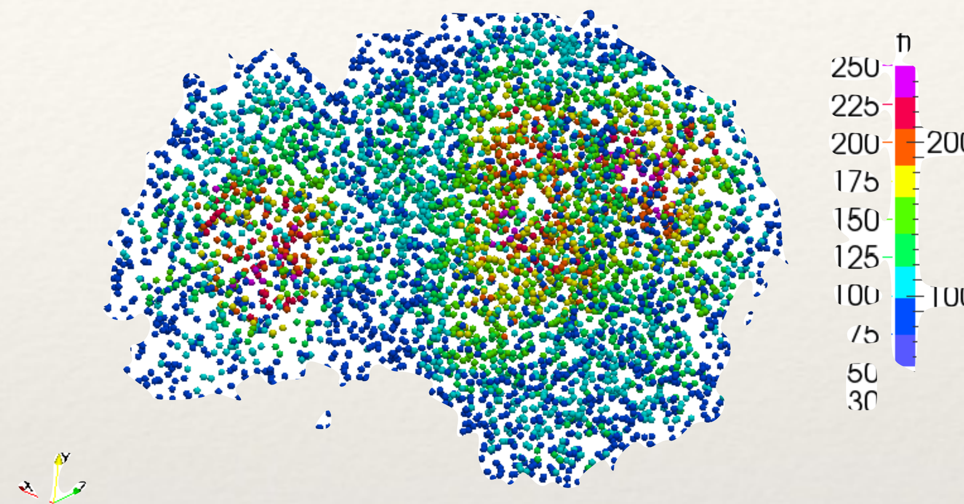
n_{ff} =15

n_{ff} = 25

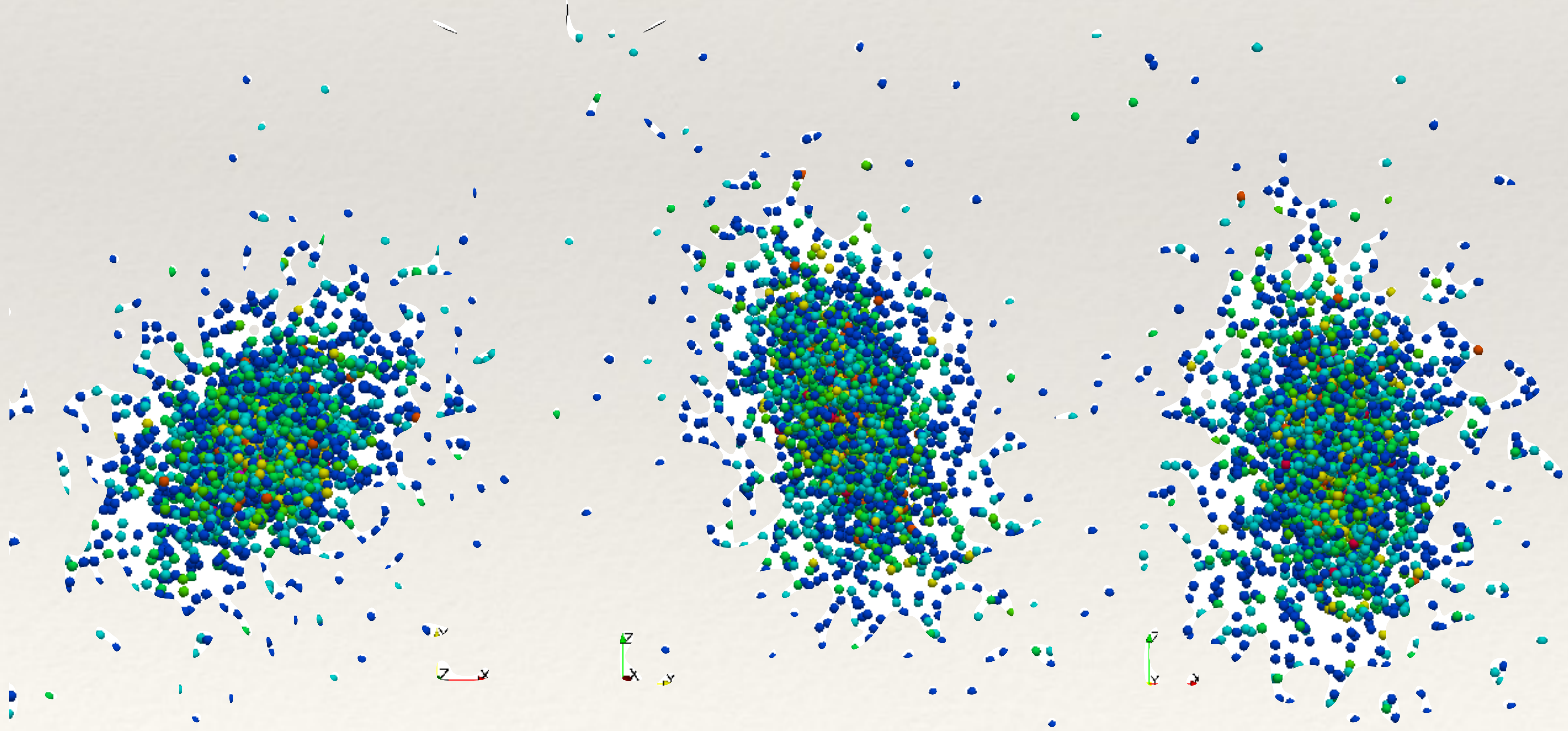
Merging halos in **q**-, **x**- & **v**-spaces

q-space

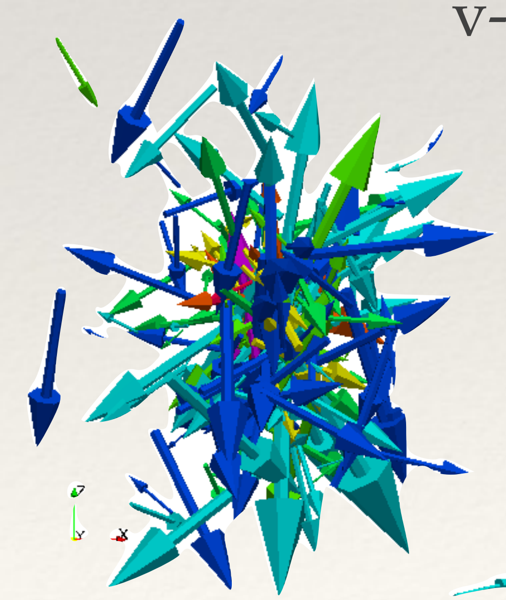




x-space



v-space



Conclusions

- **x = x(q) — Lagrangian submanifold**
  - dynamically equivalent to phase space
  - superior than phase space:
    - *single-valued mapping* — epimorphism: $\mathbf{q} \mapsto \mathbf{x}$
    - *metric space*
    - *algebraic identification of structures:* voids, walls, filaments, halos, streams
    - *no 3D projection effects:* disentangle substructure individually
    - *much more accurate*
- **Simulations + x(q) + "flip-flop field"**:
  - hierarchical structure of Cosmic Web — *a la* matryoshka doll
  - substructure survives for a long time
- "**Topological cosmology**" or "**Diophantine cosmology**" — integer numbers involved