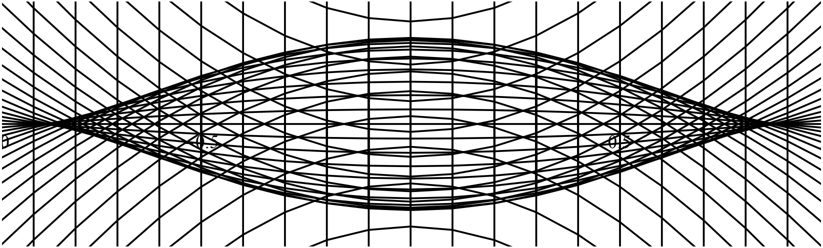
Statistics of Caustics in Large-Scale Structure Formation



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Large-Scale structure

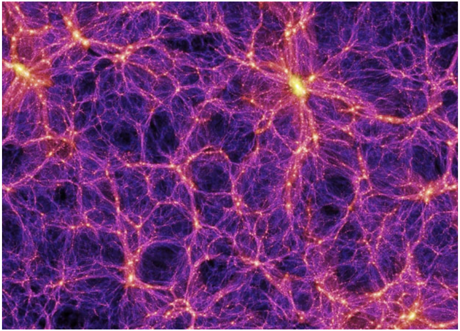
Model evolution as a DM fluid

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot (1+\delta)\mathbf{v} = 0,$$
$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{a}(\mathbf{v}\cdot\nabla_x)\mathbf{v} + \frac{\dot{a}}{a}\mathbf{v} = -\frac{1}{a}\nabla\phi,$$
$$4\pi G a^2 \rho_u \delta = \nabla_x^2 \phi.$$

with

$$\mathbf{x} = \frac{\mathbf{r}}{a}, \delta + 1 = \frac{\rho}{\rho_u},$$
$$\mathbf{v} = a\dot{\mathbf{x}}, \phi = \Phi - \frac{1}{2}a\ddot{a}\mathbf{x}^2$$

Cosmic-Web: Millennium Simulation



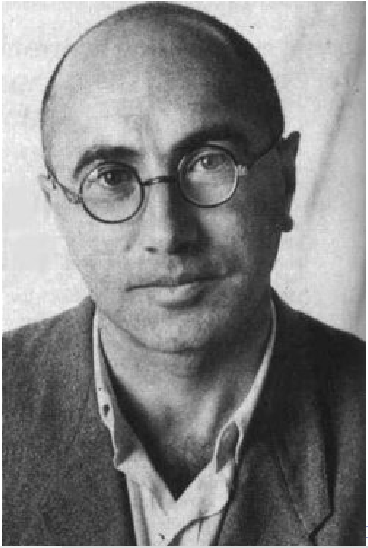
Zel'dovich Approximation

The Lagrangian approach

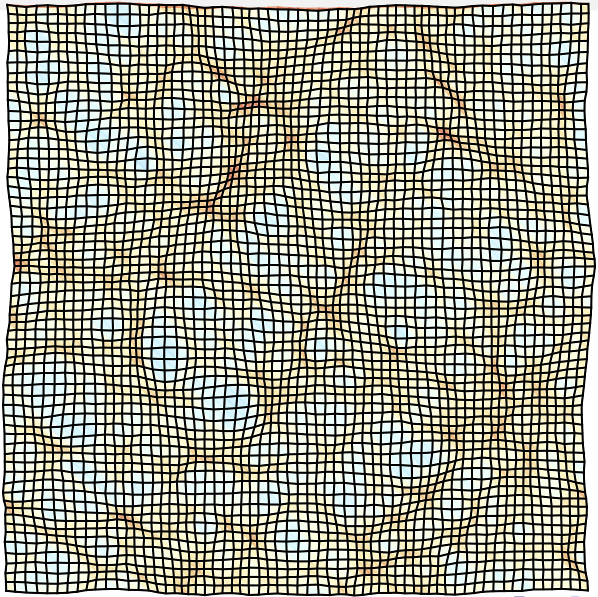
$$(\mathbf{q}, t) \mapsto \mathbf{x}(\mathbf{q}, t) = \mathbf{q} + \mathbf{s}(\mathbf{q}, t).$$

Zel'dovich approximation (1970)

$$\mathbf{x} = \mathbf{q} + D_+ \mathbf{u} = \mathbf{q} - D_+ \nabla \Psi_0,$$



Zel'dovich Approximation



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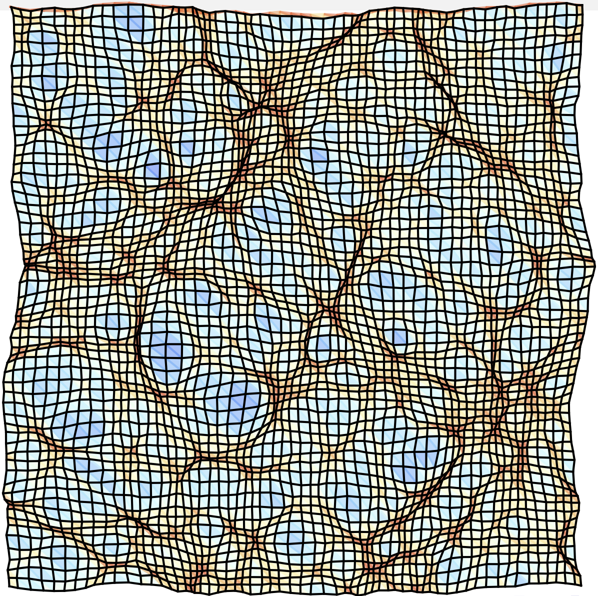

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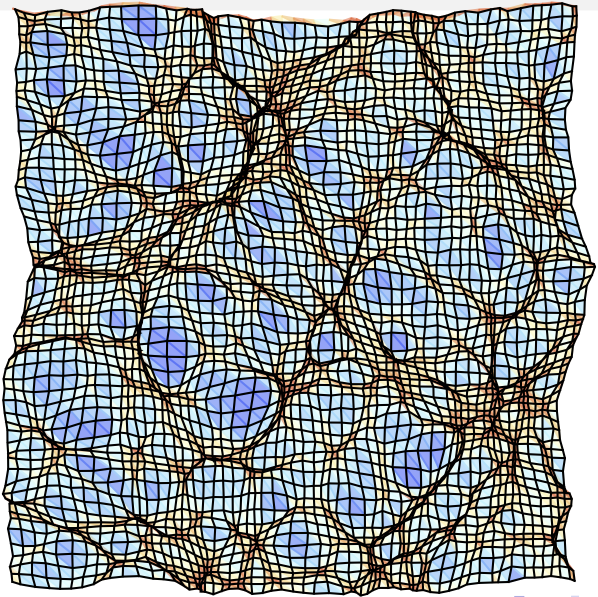


Zel'dovich Approximation

The density is

$$\rho = 1 + \delta = \left\| \frac{\partial \mathbf{x}}{\partial \mathbf{q}} \right\|^{-1} = \frac{1}{(1 - D_+\lambda_1)(1 - D_+\lambda_2)}$$

with ordered eigenvalues $\lambda_1 \geq \lambda_2$ of deformation tensor

$$\psi_{ij} = \frac{\partial^2 \Psi_0}{\partial q_i \partial q_j}.$$

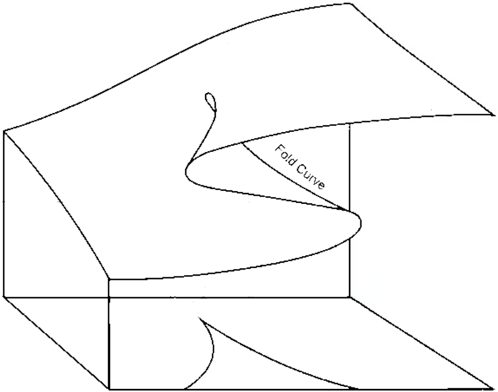
Cosmic Phase-Space and Catastrophe Theory

Lagrangian and Eulerian position $\mathbf{q}$ and $\mathbf{x}$, Phase-Space $(\mathbf{q}, \mathbf{x})$

Lagrangian catastrophe theory

- Folds A_2
- Cusps A_3
- Swallowtail A_4
- Umbilic D_4

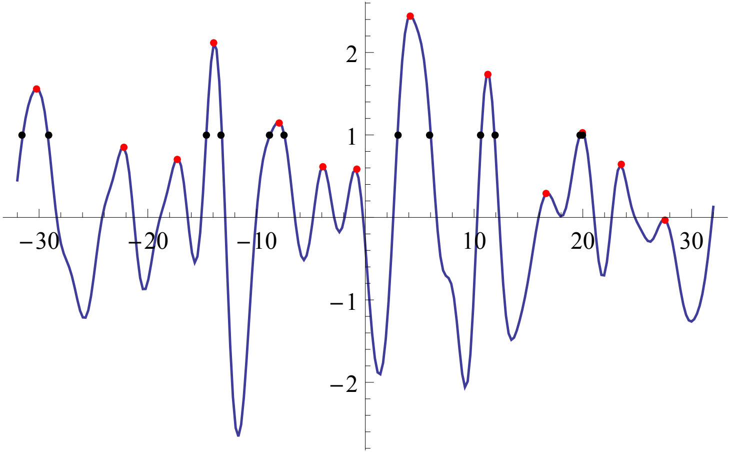
Arnol'd (1972)



Cosmic Phase-Space and Catastrophe Theory

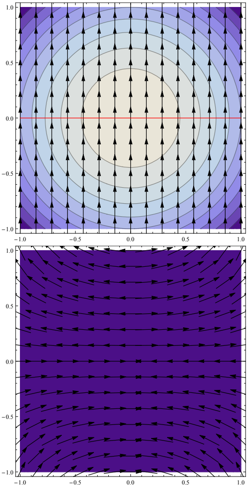
Caustic conditions on the Zel'dovich approximation

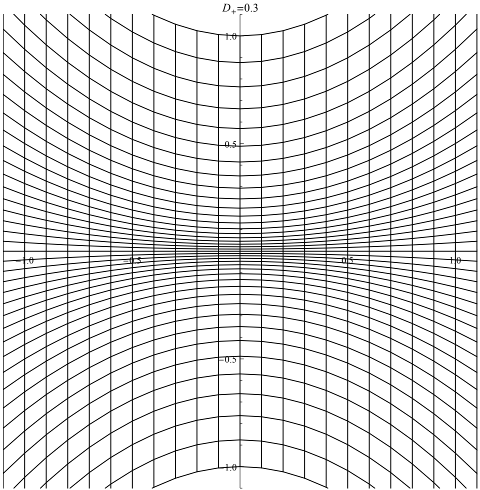
Catastrophes: $A_2, A_3^{\pm}$



Caustic conditions on the Zel'dovich approximation

Catastrophes: $A_2, A_3^{\pm}, A_4, D_4^{\pm}$, Aronl'd, Shandarin, Zel'dovich (1982)





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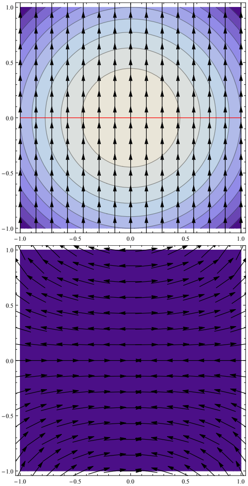

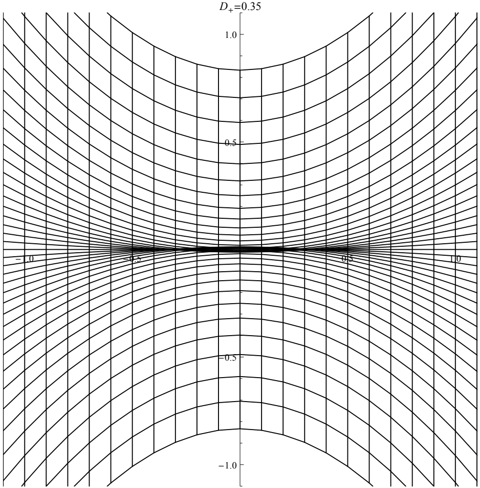

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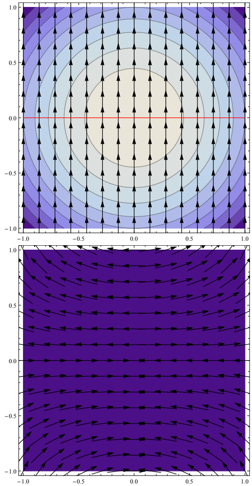
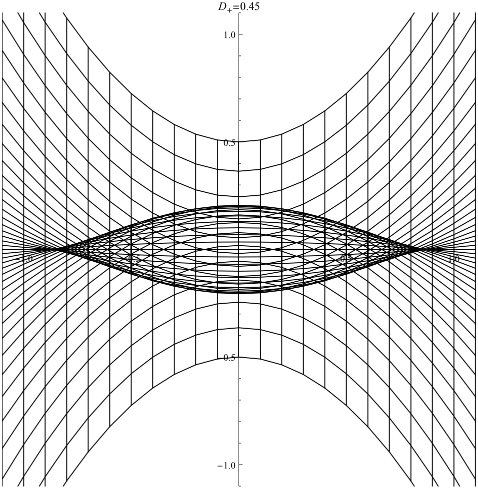




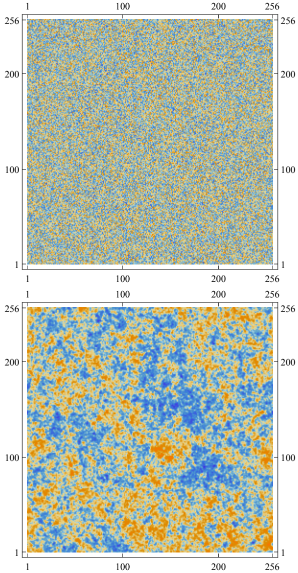
Primordial Initial Conditions: Gaussian Random Field

Probability distribution of a Gaussian Random Field

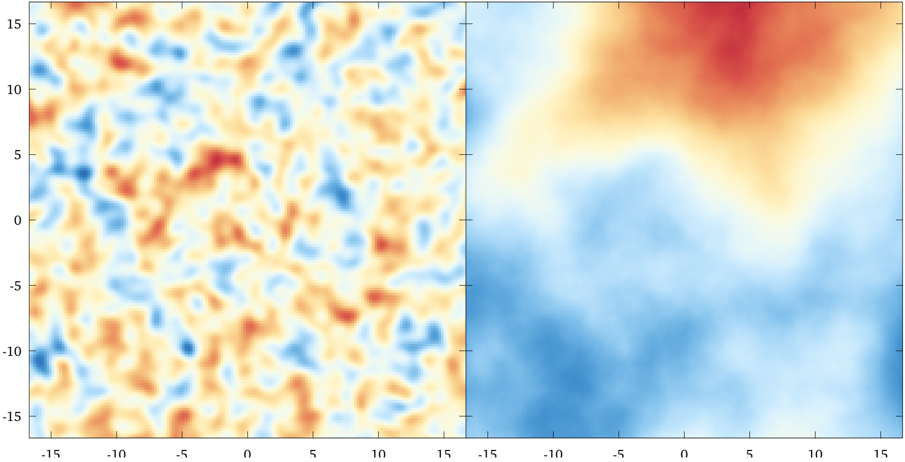
$$P(f(t_1),\dots,f(t_n)) = \frac{e^{-\frac{1}{2}\sum_{i,j} f(t_i)(M^{-1})_{ij} f(t_j)}}{[(2\pi)^n \det M]^{1/2}} \mathrm{d}f(t_1)\dots\mathrm{d}f(t_n)$$

for all $(t_1,\dots,t_n) \in T^n$, with $M_{ij} = \langle f(t_i) f(t_j)\rangle$.

Adler and Taylor (2007)
BBKS (1986)

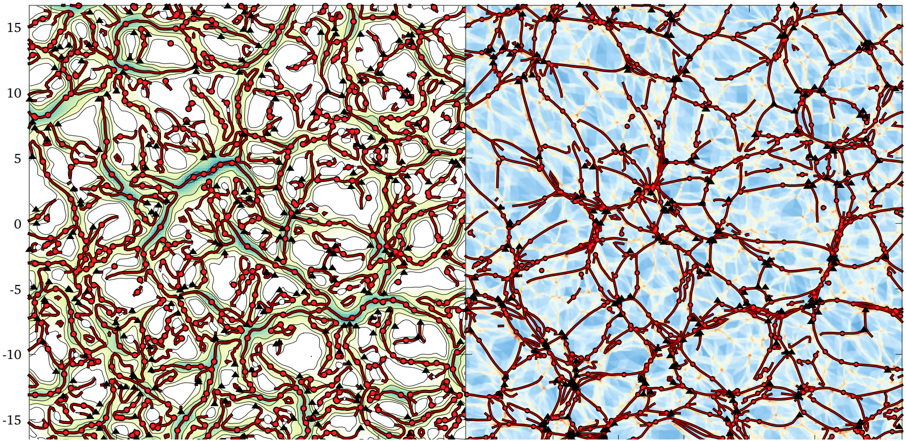


Caustic conditions on the Zel'dovich approximation



Hidding PhD Thesis (2014) (in preparation)
Feldbrugge, Hidding, vd W. et al. (2014) (in preparation)

Caustic conditions on the Zel'dovich approximation



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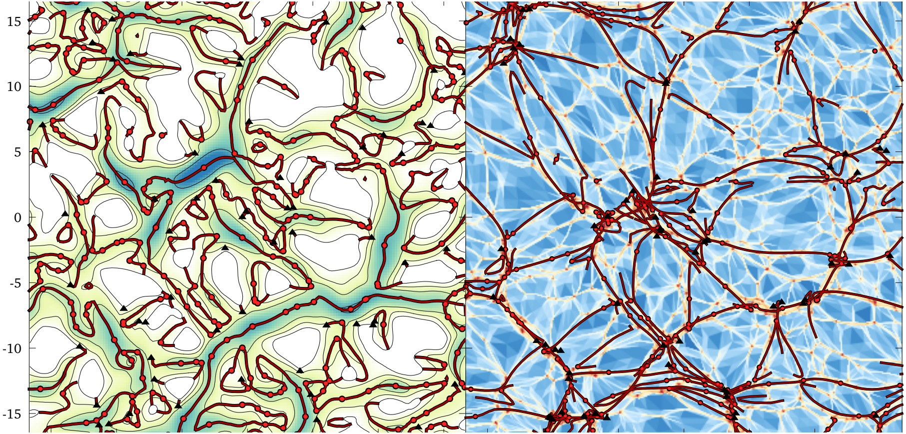

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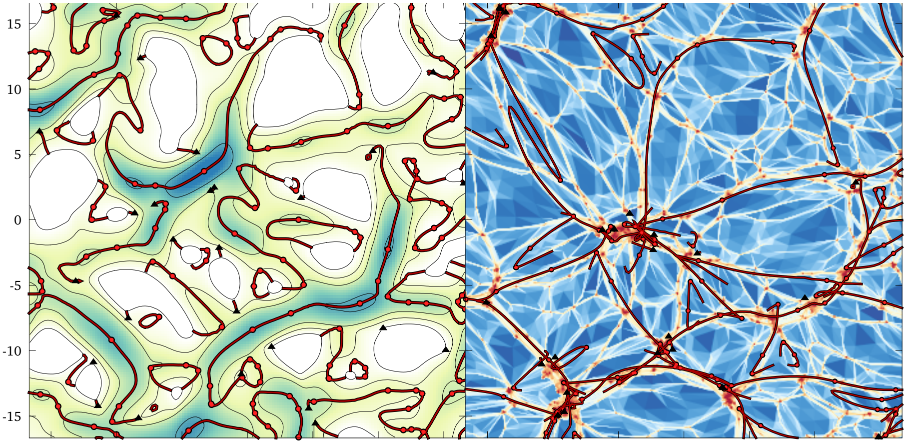

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Caustics Statistics: 1D

Level crossing $f = \lambda$

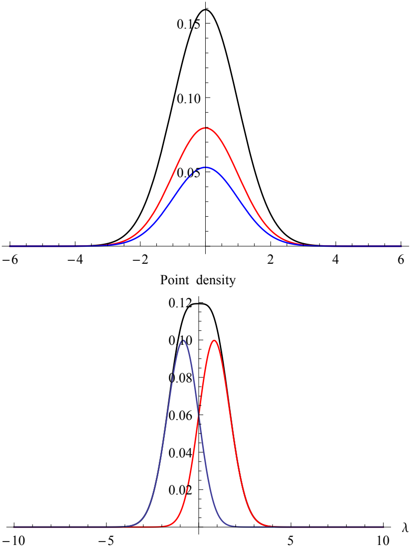
$$\mathcal{N} = \int |\dot{f}| p(f = \lambda, \dot{f}) \mathrm{d}\dot{f}.$$

$A_2, A_3^{\pm}$ point density

$$\mathcal{N}_{A_2}(\lambda) = \int |\lambda_{11}| p(\lambda_1 = \lambda, \lambda_{11}) \mathrm{d}\lambda_{11}$$

$$\mathcal{N}_{A_3^{\pm}}(\lambda) = \int |\lambda_{111}| \, p(\lambda_1 = \lambda, \lambda_{11} = 0, \lambda_{111}) \mathrm{d}\lambda_{111}$$

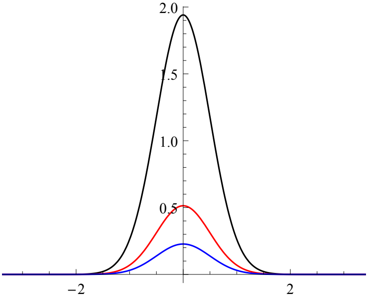
S. Rice (1944)



Caustics Statistics: 2D

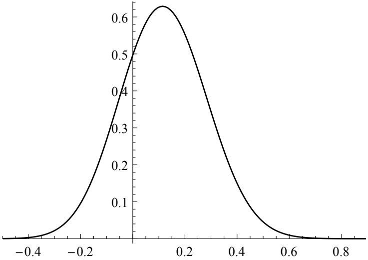
D_4 point density

$$\mathcal{N}_{D_4}(\lambda) = \int |\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}| \, p(\lambda_1 = \lambda, \lambda_2 = \lambda, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}) \times \mathrm{d}\lambda_{11}\mathrm{d}\lambda_{12}\mathrm{d}\lambda_{21}\mathrm{d}\lambda_{22}$$



A_3 lines

$$\mathcal{L}_{A_3}(\lambda_1) = \pi \int \sqrt{\lambda_{111}^2 + \lambda_{112}^2} \, p(\lambda_1, \lambda_2, \lambda_{11} = 0, \lambda_{111}, \lambda_{112}) \times (\lambda_1 - \lambda_2)\mathrm{d}\lambda_{111}\mathrm{d}\lambda_{112}\mathrm{d}\lambda_2$$



M.S. Longuet-Higgins (1957)

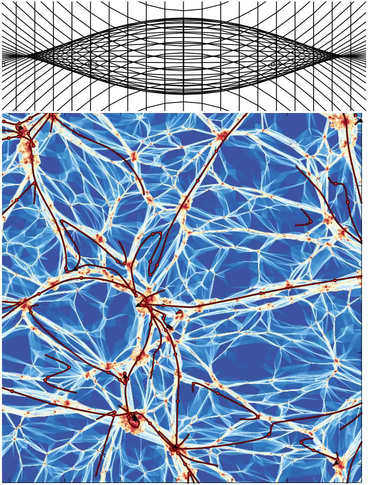
Conclusions

Summary

- Cosmic web closely linked to caustics
- Caustics classification of cosmic web
- Web allows analytic statistical analysis

Further research

- Two-point correlation functions
- Generalization to 3D
- Beyond Zel'dovich approximation



Lagrangian normal forms

At every point, the germs of generic Lagrangian maps of manifolds of dimension $n \leq 5$ are equivalent to germs of projetions $(p, q) \mapsto q$ of Lagrangian manifolds $p_I = \partial S / \partial q_I, q_J = -\partial S / \partial p_J$, where

for $n \geq 1$	$A_1 : S = p_1^2$
	$A_2 : S = p_1^3$
for $n \geq 2$ also	$A_3 : S = \pm p_1^4 + q_2 p_1^2$
for $n \geq 3$ also	$A_4 : S = p_1^5 + q_2 p_1^3 + q_3 p_1^2$
	$D_4 : S = p_1^3 \pm p_1 p_2^2 + q_3 p_1^2$