DE LA RECHERCHE À L'INDUSTRIE

NON-LINEAR DESCRIPTION OF MASSIVE NEUTRINOS IN THE FRAMEWORK OF LARGE-SCALE STRUCTURE FORMATION

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[arXiv: 1311.5487]

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The standard **linear** description of **neutrinos**

- The Vlasov equation: $$\frac{\mathrm{d}f}{\mathrm{d}\eta} = 0.$$

time coordinate

- Decomposition of the phase-space distribution function:

$$f = f_0(1 + \Psi).$$

homogeneous part

first order perturbation

- Evolution equation of $\tilde{\Psi} \equiv \left(\frac{\mathrm{d}\log f_0(q)}{\mathrm{d}\log q}\right)^{-1}\Psi$ (with $\alpha \equiv \frac{\vec{k}.\vec{q}}{kq}$):

$$\partial_\eta \tilde{\Psi} + \mathrm{i}\alpha k \frac{q}{a\epsilon}\tilde{\Psi} + \partial_\eta \phi - \mathrm{i}\alpha k \frac{a\epsilon}{q}\psi = 0.$$

momentum

energy

metric perturbations

The standard **linear** description of **neutrinos**

- Decomposition into Legendre polynomials:

$$\tilde{\Psi} = \sum_{\ell} (-\mathrm{i})^{\ell} \tilde{\Psi}_{\ell} \, P_{\ell}(\alpha).$$

- It leads to the standard Boltzmann hierarchy

$$\partial_\eta \tilde{\Psi}_0(\eta, q) = -\frac{qk}{3a\epsilon} \tilde{\Psi}_1(\eta, q) - \partial_\eta \phi(\eta)$$

$$\partial_\eta \tilde{\Psi}_1(\eta, q) = \frac{qk}{a\epsilon} \left(\tilde{\Psi}_0(\eta, q) - \frac{2}{5} \tilde{\Psi}_2(\eta, q) \right) - \frac{a\epsilon k}{q} \psi(\eta),$$

$$\partial_\eta \tilde{\Psi}_\ell(\eta, q) = \frac{qk}{a\epsilon} \left[\frac{\ell}{2\ell - 1} \tilde{\Psi}_{\ell-1}(\eta, q) - \frac{\ell + 1}{2\ell + 3} \tilde{\Psi}_{\ell+1}(\eta, q) \right] \quad (\ell \geq 2).$$

Attempts to get a **non-linear** description of **neutrinos**

Why not computing a non-linear hierarchy?

Introducing $A^{ij\ldots k} \equiv \int \mathrm{d}^3\mathbf{q} \left[\frac{q^i}{a\epsilon}\frac{q^j}{a\epsilon}\ldots\frac{q^k}{a\epsilon}\right]\frac{\epsilon f}{a^3},$

the non-linear moments of the Vlasov equation are

$$\partial_\eta A^{i_1\ldots i_n} + (\mathcal{H} - \partial_\eta\phi)\left[(n+3)A^{i_1\ldots i_n} - (n-1)A^{i_1\ldots i_n jj}\right]$$
$$+ \sum_{m=1}^{n}(\partial_{i_m}\psi)A^{i_1\ldots i_{m-1} i_{m+1}\ldots i_n} + \sum_{m=1}^{n}(\partial_{i_m}\phi)A^{i_1\ldots i_{m-1} i_{m+1}\ldots i_n jj}$$
$$+(1+\phi+\psi)\partial_j A^{i_1\ldots i_n j} + \left[(2-n)\partial_j\psi - (2+n)\partial_j\phi\right]A^{i_1\ldots i_n j} = 0.$$

See Nicolas Van de Rijt's PhD thesis "*Signatures of the primordial universe in large-scale structure surveys*", École Polytechnique & IPhT, CEA Saclay, 2012.

The **non-linear** description of **cold dark matter**: a model to follow

- Cold dark matter: collection of identical point particles, non-relativistic, sensitive to gravitational interaction only.

- Physics is encoded in the Vlasov-Poisson system (newtonian approximation).

- The Vlasov-Poisson system leads to the continuity and Euler equations:

$$\frac{\partial\delta(\mathbf{x},t)}{\partial t}+\frac{1}{a}[(1+\delta(\mathbf{x},t))u_i(\mathbf{x},t)]_{,i}=0,$$

$$\frac{\partial u_i(\mathbf{x},t)}{\partial t}+\frac{\dot{a}}{a}u_i(\mathbf{x},t)+\frac{1}{a}u_j(\mathbf{x},t)u_i(\mathbf{x},t)_{,j}=-\frac{1}{a}\Phi(\mathbf{x},t)_{,i}-\frac{(\rho(\mathbf{x},t)\sigma_{ij}(\mathbf{x},t))_{,j}}{a\rho(\mathbf{x},t)}.$$

density contrast velocity field gravitational potential velocity dispersion

The **non-linear** description of **cold dark matter**: a model to follow

- Particles are “cold” thus velocity dispersion is negligible:

$$\cancel{\frac{(\rho(\mathbf{x},t)\sigma_{ij}(\mathbf{x},t))_{,j}}{a\rho(\mathbf{x},t)}}\,.$$

- This is called the single-flow approximation.

- Note: the single-flow approximation breaks down when shell-crossing occurs, i.e. when gravity makes several flows appear (see the picture below).

The **non-linear** description of **cold dark matter**: a model to follow

In the single-flow approximation, the Euler equation reads

$$\frac{\mathrm{d}(a u_i(\mathbf{x},t))}{\mathrm{d}t} = -\Phi(\mathbf{x},t)_{,i}.$$

The velocity field can entirely be described by its divergence:

$$\theta(\mathbf{x},t) = 1/(aH) u_i(\mathbf{x},t)_{,i}.$$

In Fourier space, equations can be written compactly:

$$\boxed{\frac{\partial \Psi_a(\mathbf{k},\eta)}{\partial \eta} + \Omega_a{}^b(\eta)\Psi_b(\mathbf{k},\eta) = \gamma_a{}^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1,\eta)\Psi_c(\mathbf{k}_2,\eta),}$$

where $\Psi_a(\mathbf{k},\eta) \equiv (\delta(\mathbf{k},\eta), -\theta(\mathbf{k},\eta)),\ \ \gamma_a{}^{bc}(\mathbf{k}_a,\mathbf{k}_b) = \gamma_a{}^{cb}(\mathbf{k}_b,\mathbf{k}_a),$

$$\gamma_2{}^{22}(\mathbf{k}_1,\mathbf{k}_2) = \delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{|\mathbf{k}_1+\mathbf{k}_2|^2\,(\mathbf{k}_1.\mathbf{k}_2)}{2k_1^2k_2^2},$$

$$\gamma_2{}^{21}(\mathbf{k}_1,\mathbf{k}_2) = \delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}_1+\mathbf{k}_2).\mathbf{k}_1}{2k_1^2}$$ and $\gamma = 0$ otherwise.

The **non-linear** description of **cold dark matter**: a model to follow

The compact equation of motion has a formal solution.

$$\Psi_a(\mathbf{k},\eta) = g_a{}^b(\eta)\Psi_b(\mathbf{k},\eta_0) + \int_{\eta_0}^{\eta} \mathrm{d}\eta' g_a{}^b(\eta,\eta')\gamma_b{}^{cd}(\mathbf{k}_1,\mathbf{k}_2)\Psi_c(\mathbf{k}_1,\eta')\Psi_d(\mathbf{k}_2,\eta').$$

initial time

Green function

See the *Les Houches Summer School* lecture notes by F.Bernardeau (arXiv 1311.2724) for more details.

A **non-linear** alternative to the standard description of **neutrinos**

- Idea: describing neutrinos as a collection of single-flow fluids in order to take advantage of the single-flow approximation.
- One fluid of the collection = the gathering of all the neutrinos that have a given velocity at initial time.
- Such fluids are actually single flows if we assume that there is no shell-crossing (which is not a reasonable assumption for the overall neutrino fluid because of velocity dispersion).

A **non-linear** alternative to the standard description of **neutrinos**

- Calculations are performed in a perturbed Friedmann-Lemaître metric

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2\psi)\,\mathrm{d}\eta^2 + (1-2\phi)\,\mathrm{d}x^i\mathrm{d}x^j\delta_{ij}\right].$$

- First motion equation: conservation of the number of particles

$$\boxed{\partial_\eta n - (1+2\phi+2\psi)\partial_i\left(\frac{P_i}{P_0}n\right) = 3n(\partial_\eta\phi - \mathcal{H}) + n(2\partial_i\psi - \partial_i\phi)\frac{P_i}{P_0}.}$$

proper number density

comoving momentum field

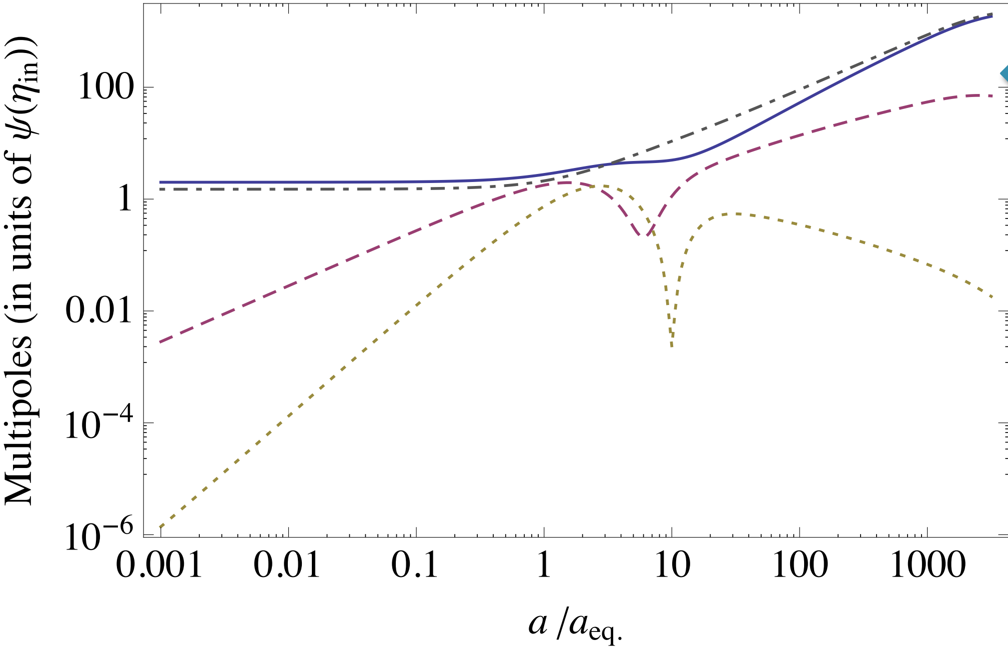
A **non-linear** alternative to the standard description of **neutrinos**

- Second motion equation: conservation of the energy-momentum tensor combined with the conservation of the number of particles

$$\boxed{\partial_\eta P_i - (1 + 2\phi + 2\psi)\frac{P_j}{P_0}\partial_j P_i = P_0 \partial_i \psi + \frac{P_j P_j}{P_0}\partial_i \phi.}$$

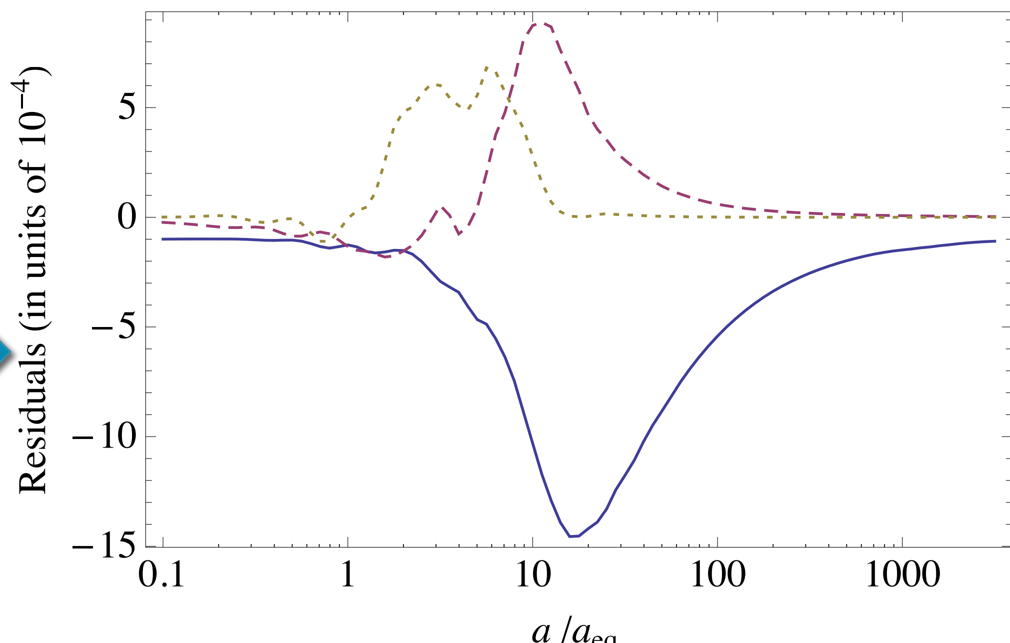
- This conservation equation makes only the momentum variable appear because of the single-flow approximation.

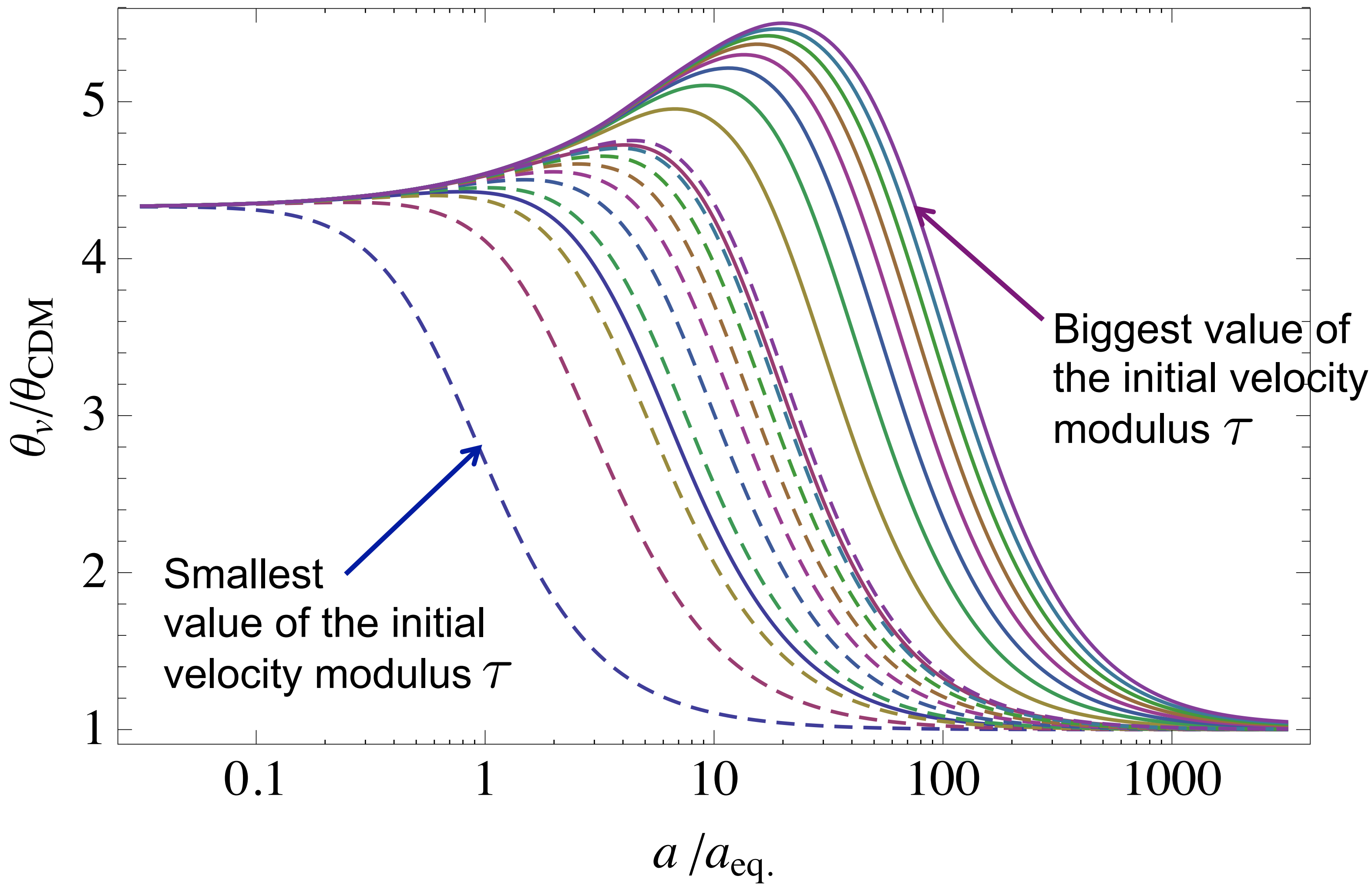
40 neutrino fluids, $k = k_{\text{eq}} = 0.01h/\text{Mpc}$, $m = 0.3$ eV



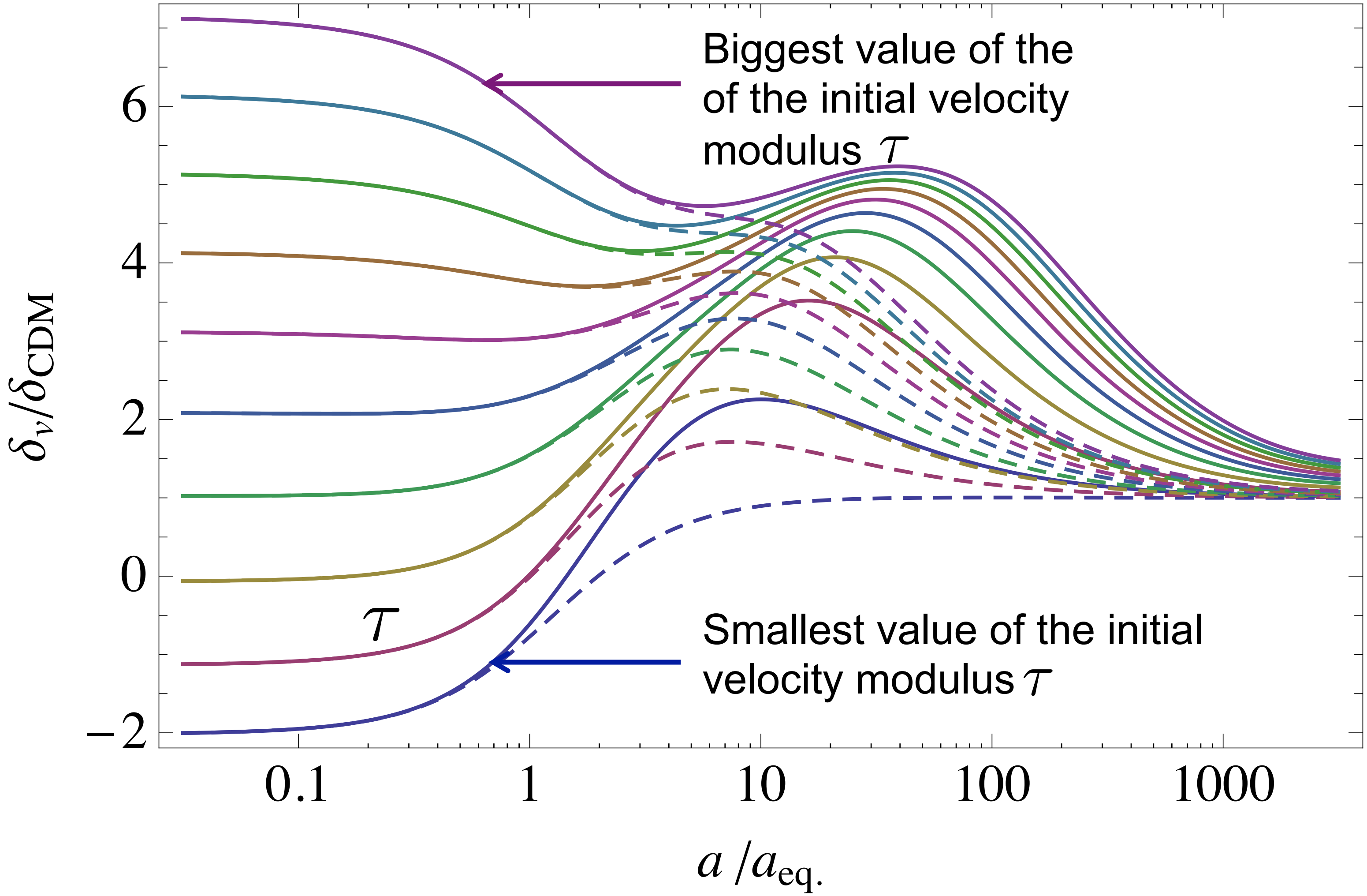
Time evolution of the energy multipoles computed using the multi-fluid description.
Solid line: energy density contrast.
Dashed line: velocity divergence.
Dotted line: shear stress.
Dot-dashed line: energy density contrast of the CDM component.

Relative differences between energy mulitpoles when they are computed in both approaches, in units of 10^{-4}.





Time evolution of the velocity divergence for different values of τ with $(\vec{k}.\vec{\tau})/(k\tau) = 0$ $(0.45 k_{\rm B} T_0 < \tau < 9 k_{\rm B} T_0)$.
Solid lines: $m = 0.05eV$. Dashed lines: $m = 0.3eV$.



Time evolution of the number density contrast for different values of τ with $(\vec{k}.\vec{\tau})/(k\tau) = 0$ $(0.45k_{\rm B}T_0 < \tau < 9k_{\rm B}T_0)$.
Solid lines: $m = 0.05eV$. Dashed lines: $m = 0.3eV$.

Conclusions

- We have developed an analytical non-linear description of massive neutrinos based on perturbation theory.
- This approach is valid up to shell-crossing only (same restriction as in the cold dark matter description).
- Developing - and then implementing - an advanced theory allowing to make predictions about relevant non-linear observables is challenging but *a priori* doable in this context.