Halo Bias and its Evolution in the Peak Model

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Motivation & Strategy

Motivation for Theoretical Modelling of Halo Clustering Statistics

- extraction of fundamental physics from galaxy clustering surveys requires accurate and precise bias models
  - BAO smoothing/reconstruction for dark energy
  - broadband power for neutrino mass
- halo-halo correlation function is important ingredient of halo model

Strategy

[Bond & Myers 1993]

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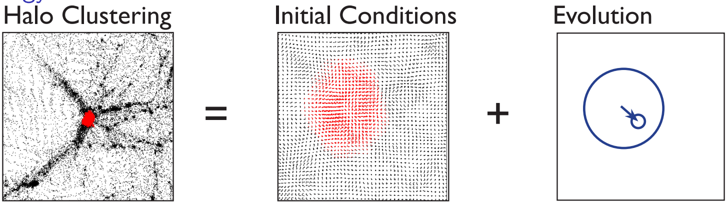


[Bond & Myers 1993]

Bias Models on the Market

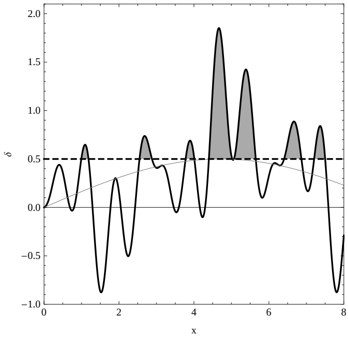
non-Perturbative models

- Kaiser thresholded regions $\delta > \delta_c$

$$\xi_{tr}(r) \xrightarrow{r\to\infty} b_1^2 \xi_{lin}(r)$$

- Peak formalism $\delta > \delta_c, \delta' = 0, \delta'' < 0$

$$\xi_{pk}(r) \xrightarrow{r\to\infty} b_1^2 (1 - \kappa_\delta \nabla^2)^2 \xi_{lin}(r)$$



Perturbative models

- local Lagrangian Bias
- local Eulerian Bias

[Kaiser 1984,BBKS 1986,Fry & Gaztanaga 1993]

1-D Peak Clustering Primer

Peak Density

$$1+\delta_{\rm pk}(\mathbf{q}) = \frac{1}{\bar{n}_{\rm pk}}\sum_{\rm pk}\delta^{\rm (D)}(\mathbf{q}-\mathbf{q}_{\rm pk}) = \frac{1}{\bar{n}_{\rm pk}}\sum_{\rm pk}\det H\delta^{\rm (D)}(\boldsymbol{\nabla}\delta).$$

Peak Correlation Function $\mathbf{Y}=(\delta_1,\delta_1',\delta_1'',\delta_2,\delta_2',\delta_2'')$

$$1+\xi_{\rm pk}(r) = \frac{1}{\bar{n}_{\rm pk}^2}\frac{1}{\sqrt{(2\pi)^6\det M}}\int_{-\infty}^{0}{\rm d}\delta_2''\,\delta_2''\;{\rm d}\delta_1''\,\delta_1''\;\exp\left[-\frac{1}{2}\mathbf{Y}M^{-1}\mathbf{Y}^T\right]$$

$$\xi_{\rm pk,pk}(r)\approx b_{10}^2\xi_0(r)+2b_{10}b_{01}\xi_1(r)+b_{01}^2\xi_2(r)$$

$$\xi_{{\rm pk},\delta}(r)=b_{10}\xi_0(r)+b_{01}\xi_1(r)$$

Correlators of Derivatives of Smoothed Field & Covariance Matrix

$$\xi_{(n+m)/2}(r)=\left\langle\delta^{(n)}(0)\delta^{(m)}(r)\right\rangle=\int\frac{{\rm d}^3k}{(2\pi)^3}(-1)^n({\rm i}\mu k)^{n+m}\exp[{\rm i}\mu kr]P_{\rm 3D}(k)W_R(k)$$

$$M=\begin{pmatrix} m & B\\ B^{\rm T} & m\end{pmatrix}\quad B=\begin{pmatrix}\xi_0(r) & -\xi_{1/2}(r) & -\xi_1(r)\\ \xi_{1/2}(r) & \xi_1(r) & -\xi_{3/2}(r)\\ -\xi_1(r) & \xi_{3/2}(r) & \xi_2(r)\end{pmatrix}\quad m=\begin{pmatrix}\sigma_0^2 & 0 & -\sigma_1^2\\ 0 & \sigma_1^2 & 0\\ -\sigma_1^2 & 0 & \sigma_2^2\end{pmatrix}$$

[Kac 1943, Rice 1954, BBKS 1986, Lumsden, Heavens & Peacock 1989]

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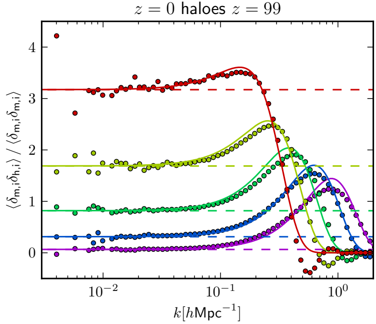
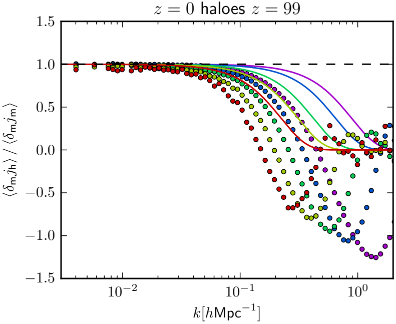
Peak Effects in the Initial Conditions

$$b_\delta(k) = \left(b_{10} + b_{01}k^2\right) W_R(k)$$

$$b_v(k) = W_R(k)$$



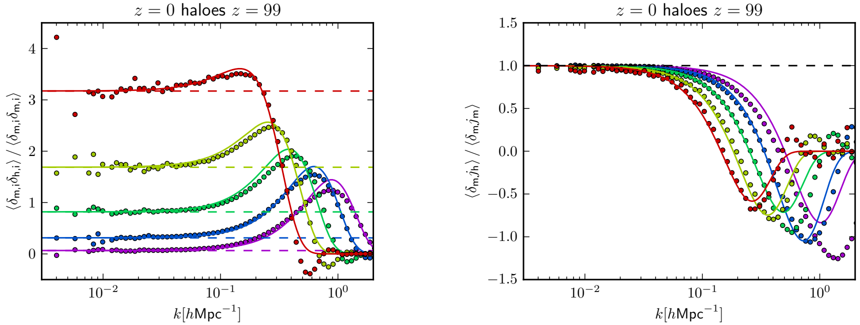


$$j(\mathbf{x}) = \left[1 + \delta(\mathbf{x})\right]\mathbf{v}(\mathbf{x}) \cdot \hat{\mathbf{e}}_z$$

[Desjacques et al. 2009, Elia et al. 2011, Baldauf et al. 2014]

Peak Effects in the Initial Conditions

$$b_\delta(k) = \left(b_{10} + b_{01} k^2\right) W_R(k) \qquad b_v(k) = \left(1 - R_v^2 k^2\right) W_R(k)$$



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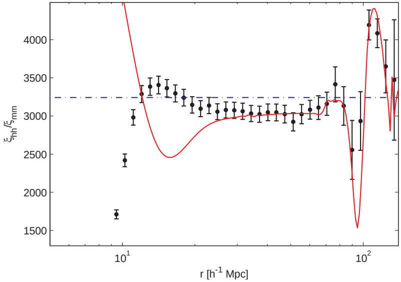
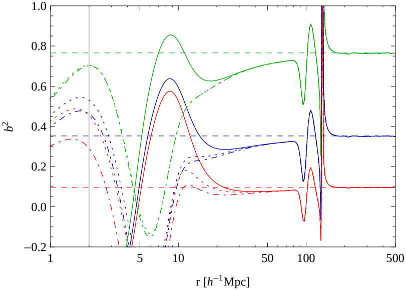
Peak bias in the Correlation Function

$$\xi_{\rm hh}(r) \xrightarrow{r\to\infty} (b_{10}-b_{01}\nabla^2)^2\xi_R(r) \;\not\Rightarrow\; P_{\rm hh} \approx (b_{10}-b_{01}k^2)^2P(k)W_R^2(k)+\frac{1}{\bar{n}}$$

Simulations



1D Peak Model



[Lumsden, Heavens & Peacock 1989, Baldauf et al. 2013]

Coupled Halo-Dark Matter Fluid - Eulerian Approach

General Evolution Equations

$$\delta'_{\mathrm{m}}(\mathbf{k}) + \theta_{\mathrm{m}}(\mathbf{k}) = -\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \alpha(\mathbf{q}_1, \mathbf{q}_2)\theta_{\mathrm{m}}(\mathbf{q}_1)\delta_{\mathrm{m}}(\mathbf{q}_2)$$

$$\theta'_{\mathrm{m}}(\mathbf{k}) + \mathcal{H}\theta_{\mathrm{m}}(\mathbf{k}) + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta_{\mathrm{m}}(\mathbf{k}) = -\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \beta(\mathbf{q}_1, \mathbf{q}_2)\theta_{\mathrm{m}}(\mathbf{q}_1)\theta_{\mathrm{m}}(\mathbf{q}_2)$$

$$\delta'_{\mathrm{h}}(\mathbf{k}) + \theta_{\mathrm{h}}(\mathbf{k}) = -\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \alpha(\mathbf{q}_1, \mathbf{q}_2)\theta_{\mathrm{h}}(\mathbf{q}_1)\delta_{\mathrm{h}}(\mathbf{q}_2)$$

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Linear Evolution for b_v

[Elia et al. 2011, Baldauf et al. 2014]

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$$\begin{aligned} \delta'_{\mathrm{m}}(\mathbf{k}) + \theta_{\mathrm{m}}(\mathbf{k}) &= 0 \\ \theta'_{\mathrm{m}}(\mathbf{k}) + \mathcal{H}\theta_{\mathrm{m}}(\mathbf{k}) + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta_{\mathrm{m}}(\mathbf{k}) &= 0 \\ \delta'_{\mathrm{h}}(\mathbf{k}) + \theta_{\mathrm{h}}(\mathbf{k}) &= 0 \\ \theta'_{\mathrm{h}}(\mathbf{k}) + \mathcal{H}\theta_{\mathrm{h}}(\mathbf{k}) + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta_{\mathrm{m}}(\mathbf{k}) &= 0 \end{aligned}$$

Linear Evolution for b_v

$$b_v(k,\eta) = 1 + \left[b_{v,\mathrm{i}}(k) - 1\right]\left[\frac{D(\eta_{\mathrm{i}})}{D(\eta)}\right]^{3/2}$$

[Elia et al. 2011, Baldauf et al. 2014]

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Linear Evolution for b_v

$$b_v(k,\eta) = b_{v,\mathrm{c}}(k) + \left[b_{v,\mathrm{i}}(k) - b_{v,\mathrm{c}}(k)\right]\left[\frac{D(\eta_{\mathrm{i}})}{D(\eta)}\right]^{3/2}$$

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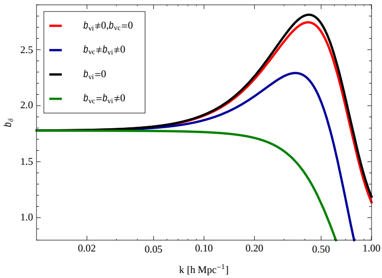
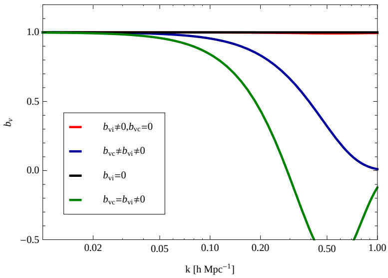
Coevolution: Possible Scenarios for late time Outcome

Density Bias



- no initial velocity bias, no coupling bias
- initial velocity bias, no coupling bias

Velocity Bias



- initial velocity bias, strong coupling bias
- initial velocity bias, weaker coupling bias

Zeldovich Evolution of Peaks

Zeldovich displaced Peaks

$$1+\delta_\mathrm{h}(\mathbf{x}) = \frac{1}{\bar{n}_\mathrm{h}} \sum_\mathrm{h} \delta^{(\mathrm{D})}(\mathbf{x}-\mathbf{x}_\mathrm{h})$$
$$= \frac{1}{\bar{n}_\mathrm{h}} \int \mathrm{d}^3 q \, \delta^{(\mathrm{D})}(\mathbf{x}-\mathbf{q}-\boldsymbol{\Psi}(\mathbf{q})) \sum_\mathrm{pk} \delta^{(\mathrm{D})}(\mathbf{q}-\mathbf{q}_\mathrm{pk})$$

Density and Momentum Statistics

$$\left\langle \delta_\mathrm{m}^{(1)}(\mathbf{k}) \delta_\mathrm{h}(-\mathbf{k}) \right\rangle = D_+^2 c_1^{(\mathrm{E})}(k,a) G_\mathrm{pk}(k) P_R(k)$$

$$\left\langle \delta_\mathrm{m}^{(1)}(\mathbf{k}) j_\mathrm{h}^z(-\mathbf{k}) \right\rangle = \left(b_v(k,a) - D_+^2 \sigma_{\mathrm{d,pk}}^2 \, c_1^{(\mathrm{E})}(k,a) \, k^2 \right) \mathcal{H} f_+ D_+^2 \left(\frac{\mathrm{i}\mu}{k} \right) G_\mathrm{pk}(k) P_R(k)$$

Ingredients

$$c_1^{(\mathrm{E})}(k,a) = 1 - R_v^2 k^2 + D_+^{-1}(b_{10} + b_{01} k^2) \qquad b_v(k,a) = 1 - R_v^2 k^2 = \mathrm{const.}$$

$$G_\mathrm{pk}(k) = \exp\left[-\frac{1}{2} \sigma_{\mathrm{d,pk}}^2 k^2 D_+^2(a) \right]$$

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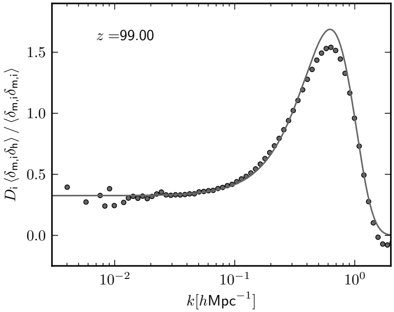
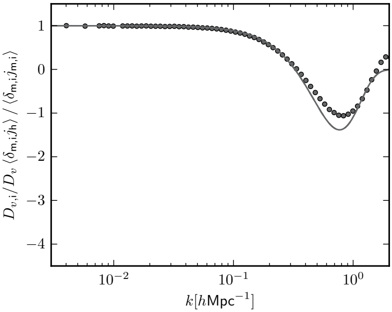
Evolution for Conserved Tracers

Density



Momentum



dashed: decaying velocity bias $R_v^2 \propto D^{-3/2}$
solid: Zeldovich displaced peaks $\sim$ constant velocity bias

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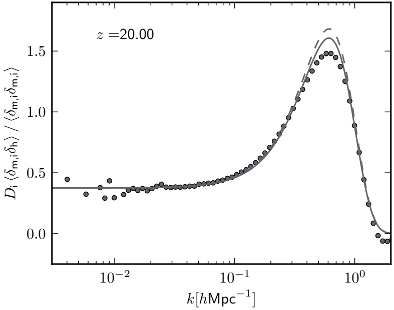

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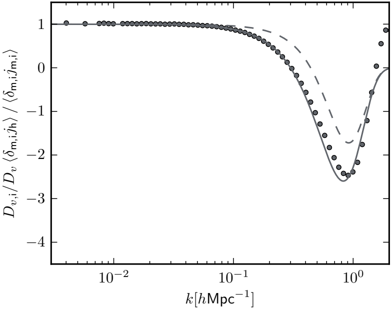

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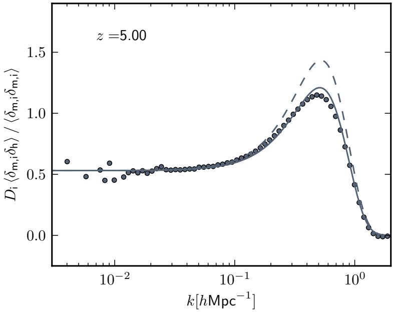

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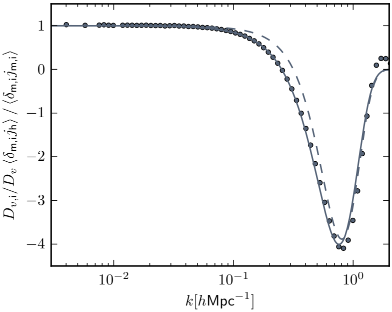

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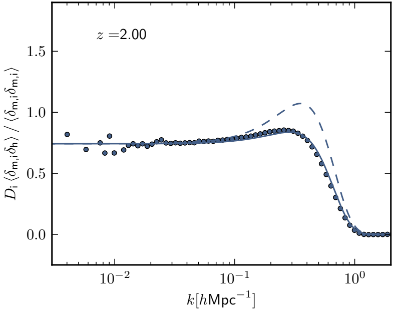

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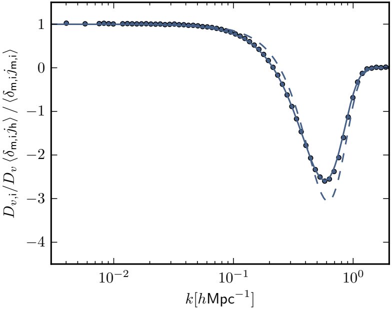

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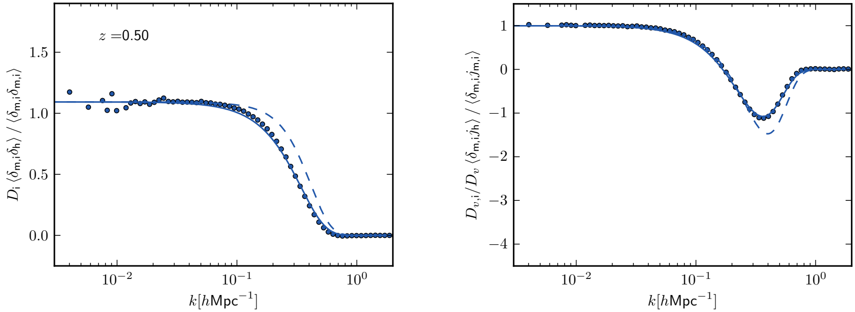

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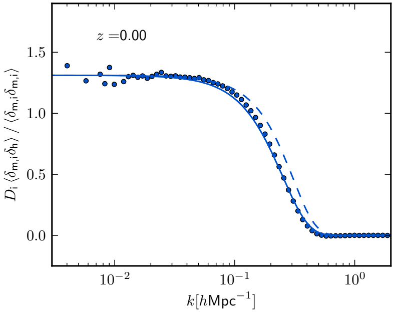

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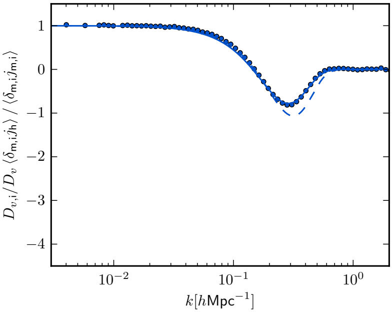

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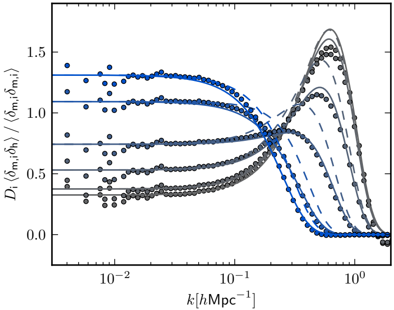

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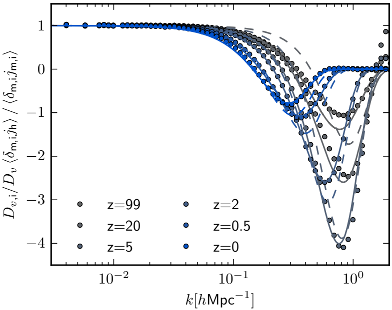

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Summary & Outlook

Findings

- evidence for peak/k^2 effects in the initial conditions
- evolution of velocity bias in tension with simplest models
- evidence for late time velocity bias $\Rightarrow$ relevant ingredient of scale dependent bias models

Ongoing work...

- late time halo-halo clustering + BAO reconstruction
- non-perturbative 3D clustering in ICs and evolved field